Digital Communications

UNIT-I

Elements of Digital Communication Systems

What Does Communication (or Telecommunication) Mean?

The term communication (or telecommunication) means the transfer of some form of information from one place (known as the source of information) to another place (known as the destination of information) using some system to do this function (known as a communication system).

Basic Terminology Used in this Communications Course

A Signal: is a function that specifies how a specific variable changes versus an independent variable such as time, location, height (examples: the age of people versus their coordinates on Earth, the amount of money in your bank account versus time).

A System: operates on an input signal in a predefined way to generate an output signal.

Analog Signals: are signals with amplitudes that may take any real value out of an infinite number of values in a specific range (examples: the height of mercury in a 10cm–long thermometer over a period of time is a function of time that may take any value between 0 and 10cm, the weight of people setting in a class room is a function of space (x and y coordinates) that may take any real value between 30 kg to 200 kg (typically)).

Digital Signals: are signals with amplitudes that may take only a specific number of values (number of possible values is less than infinite) (examples: the number of days in a year versus the year is a function that takes one of two values of 365 or 366 days, number of people sitting on a one-person chair at any instant of time is either 0 or 1, the number of students registered in different classes at KFUPM is an integer number between 1 and 100).

Noise: is an undesired signal that gets added to (or sometimes multiplied with) a desired transmitted signal at the receiver. The source of noise may be external to the communication system (noise resulting from electric machines, other communication systems, and noise from outer space) or internal to the communication system (noise resulting from the collision of electrons with atoms in wires and ICs).

SNR: Signal to noise ratio is the ratio of the power of the desired signal to the power of the noise signal.

Bandwidth : is the width of the frequency range that the signal occupies. For example the bandwidth of a radio channel in the AM is around 10 kHz and the bandwidth of a radio channel in the FM band is 150 kHz.

Rate of Communication:is the speed at which DIGITAL information is transmitted. The maximum rate at which most of today's modems receive digita' information is around 56 k bits/second and transmit digital information is around 33 k bits/second. A Local Area Network (LAN) can theoretically receive/transmit information at a rate of 100 M bits/s. Gigabit networks would be able to receive/transmit information at least 10 times that rate.

Modulation: is changing one or more of the characteristics of a signal (known as thecarrier signal) based on the value of another signal (known as the information or modulat***ing*** signal) to produce a modulat***ed*** signal.

Digital Communication Systems

The term digital communication covers a broad area of communications techniques, including digital transmission and digital radio. Digital transmission, is the transmitted of digital pulses between two or more points in a communication system. Digital radio, is the transmitted of digital modulated analog carriers between two or more points in a communication system.

Why Digital

There are many reasons

- The primary advantage is the ease with which digital signals, compared to analog signal, are regenerative.

The shape of the waveform is affected by two mechanisms:

(1) As all the transmission lines and circuits have some non-ideal transfer function, there is a distorting effect on the ideal pulse.

(2) Unwanted electrical noise or other interference further distorts the pulse waveform.

Both of these mechanisms cause the pulse shape to degrade as a function of distance.
During the time that the transmitted pulse can still be reliably identified, the pulse is thus regenerated. The circuit that perform this function at regular intervals along a transmission system are called regenerative repeaters.

- Digital circuits are less subject to distortion and interference than analog circuits.
- Digital circuits are more reliable and can be produced at lower cost than analog circuits. Also, digital hardware lends itself to more flexible implementation than analog hardware.
- Digital techniques lend themselves naturally to signal processing functions that protect against interference and jamming.
- Much data communication is computer to computer, or digital instrument or terminal to computer. Such digital terminations are naturally best served by digital link.

Communication System Models

Generally, there are two types for communication system models, ***base-band*** model and ***pass-band*** model.

In base-band model, the spectrum of signal from zero to some frequency (i.e. carrier frequency=0). For transmission of base-band signal by a digital communication system, the information is formatted so that it is represented by digital symbols. Then, pulse waveforms are assigned that represented these symbols. This step referred to as ***pulse modulation*** or ***base-band modulation***. These waveforms can be transmitted over a cable. Base-band signal also called ***low-pass signal***.

In pass-band (or band-pass) signal, the signal has a spectral magnitude that is nonzero for frequency in some band concentrate about a frequency $f=\pm f_C$ and negligible elsewhere, where fc is the carrier frequency need to be much greater than zero. For radio transmission the carrier is covered to an electromagnetic (EM) filed for propagation to desired destination.

Multiplexing

Multiplexing is the transmission of information (either voice or data) from more than one source to more than one destination on the same transmission medium.

Two most common methods are used, ***frequency division multiplexing (FDM) and time division multiplexing (TDM).***

- **FDM**

In FDM multiple sources that originally occupied the same frequency spectrum are each converted to a different frequency band and transmitted simultaneously over a single transmission medium. FDM is an analog multiplexing scheme.

- **TDM**

With TDM system, transmission from multiple sources occurs on the same transmission medium but not at the same time. Transmission from various sources is in time.

Elements of digital communication systems.

Examples of Digital Communication system:

1.E-mail:Computer

2.Text/sms: Cellphones

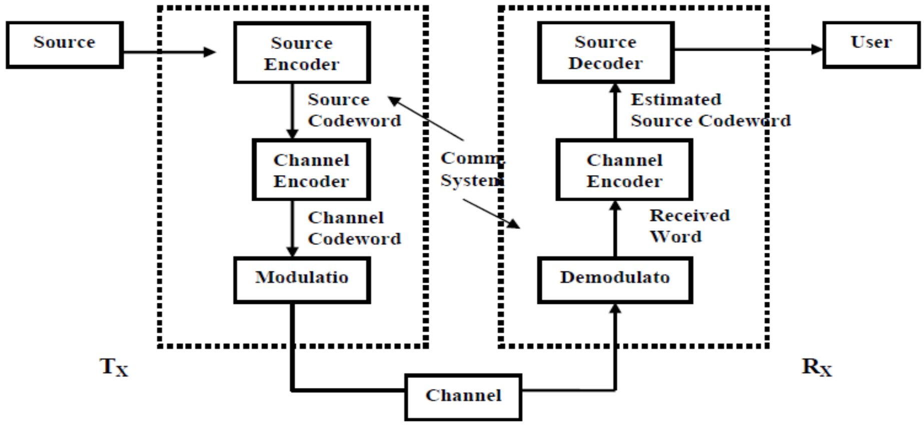
3.Fax

4.Tele-conferencing

5.Video-Conferencing

Block Schematic Description of a Digital Communication System

In the simplest form, a transmission-reception system is a three block system, consisting of a) a transmitter, b) a transmission medium and c) a receiver



Basic block diagram of a digital communication System

Digital communications:

There are two modes of communications

i) Analog mode of communication.

ii) Digital mode of communication.

$\boxed{M = 2^k}$ M array coding

k → no. of bit used to encode the sample value

M → no. of symbol

Discrete information source: The source of information can be analog or digital, e.g. analog: audio or video signal, digital: like teletype signal. In digital communication the signal produced by this source is converted into digital signal consists of 1′s and 0′s.

→It is a device which provides information in discrete form.

→Based on the nature of o/p message signal information sources are broadly classified into two types.

1. Continuous information source

   Ex: mic

2. Discrete information source

   Ex: computer

Sources encoder: It is a device which implements source encoding techniques. Source encoding technique develops/constructs a source code word for each source message. It generates source code word of length k – bits which is equivalent to the input message.

Source encoding techniques are broadly classified as:

1) Fixed length source encoding technique

2) Variable length source encoding technique

The objective of source encoding is to reduce redundancy.

Source coding techniques are to increase average information per bit. Source encoding tries to compress the data that it is receiving from the digital source into minimum number of bits. S*ource Encoding or Data Compression* is the process of efficiently converting the output of either analog or digital source into a sequence of binary digits

Variable length source encoding technique:

Example 1:A message source generates eight symbols with probability , $P(X_1)$ =1/2, $P(X_2)$ =1/4, $P(X_3)$ =1/8 , $P(X_4)$ =1/16, $P(X_5)$ =1/32, $P(X_6)$ =1/64 , $P(X_7)$ =1/128 $P(X_8)$ =1/128. Encode the message Xi with variable length binary codes using Shannon – fanon procedure.

Procedure: - Construction / development of source code words for the gives set of messages using shanon-fano procedure.

-Arrange the given set of message in the order of decreasing probability.

message	Probability
X_1	½
X_2	¼
X_3	1/8
X_4	1/16
X_5	1/32
X_6	1/64
X_7	1/128
X_8	1/128

- Divide the given set of message into two subsets such that their probabilities are equal

Message	probability								
x1	1/2	0							
x2	1/4	1	0						
x3	1/8	1	1	0					
x4	1/16	1	1	1	0				
x5	1/32	1	1	1	1	0			
x6	1/64	1	1	1	1	1	0		
x7	1/128	1	1	1	1	1	1	0	
x8	1/128	1	1	1	1	1	1	1	1

message	probability	Sources code word	length
X_1	1/2	0	1
X_2	1/4	1 0	2
X_3	1/8	1 1 0	3
X_4	1/16	1 1 1 0	4
X_5	1/32	1 1 1 1 0	5
X_6	1/64	1 1 1 1 1 0	6
X_7	1/128	1 1 1 1 1 1 0	7
X_8	1/128	1 1 1 1 1 1 1 0	8

→The main objective of variable length source encoding technique is to increase average information per bit by means of reducing redundancy.

Example 2: Construct source code word for the given set of messages using fixed length source encoding techniques.

Procedure: Fixed length source encoding technique for the same problem solution is

message	probability	Sources code word	length
X_1	1/2	000	1
X_2	1/4	001	2
X_3	1/8	010	3
X_4	1/16	011	4
X_5	1/32	100	5
X_6	1/64	101	6
X_7	1/128	110	7
X_8	1/128	111	8

Example 3: A discrete information source emits 3 messages i.e., E, F,G with the probabilities ½ , ¼, ¼ respectively. Construct source code words using shanon – Fano source encoding algorithm

Procedure:

Message	**probability**	**Source code word**	**length**
X_1 = E	½	0	1
X_2 = F	¼	1 0	2
X_3 = G	¼	1 1	2

Channel encoder: Channel encoder transforms message block of k- message bits into a code block of n bits(n>k) or Channel encoder takes a block of k- information bits from source encoder & adds "r" error control bits (check bits/ parity bits / Extra bits) to generate block code of n bits

$$N = K + r$$

→ Parity bits helps the receiver in the process of error detection & error correction.

→ Channel encoding techniques are

1) Block codes

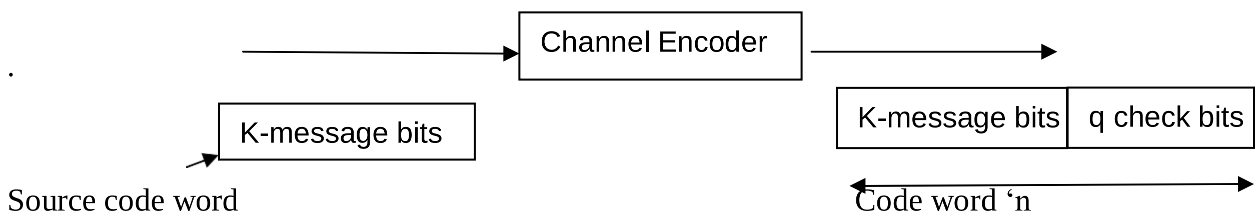
2) Cyclic codes

3) convolution codes

where n=code word length

k =message bits

q= number of check bits/parity bits



BLOCK CODE:

(n , k)

$[C]_{1\times n} = [D]_{1\times k}\,[G]_{k\times n}$

Matrix description of linear block codes

Digital signal transmission:

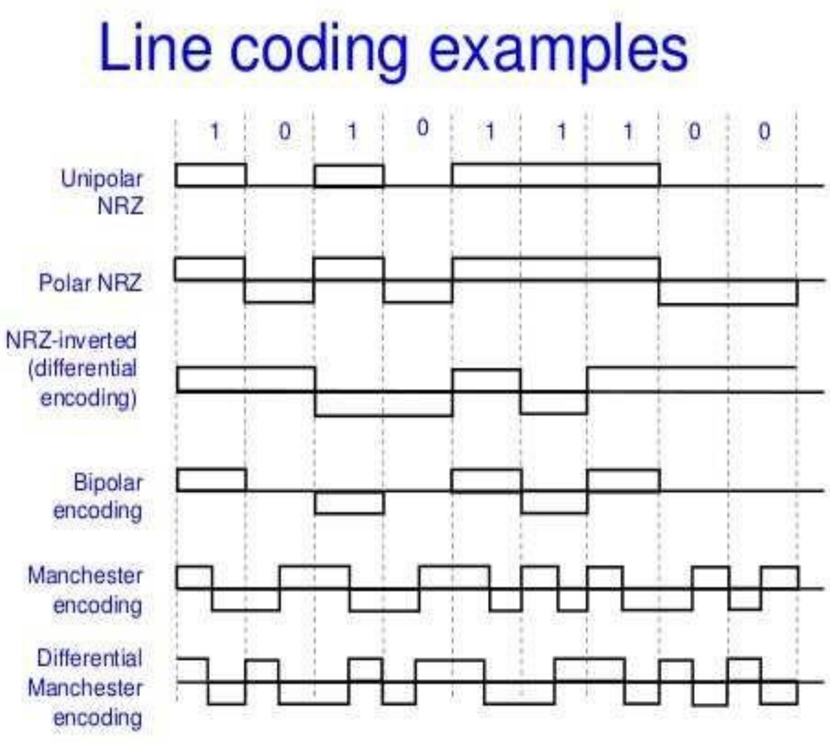
Line code:

→ in base band transmission best way is to digits (or) symbols into pulse wave form. This wave form is generally termed as " line code"

R_z : Return to zero (pulse for half duration T_b)

NRZ: non return to zero

(pulse for full duration T_b)



Modulator: The binary sequence is passed to digital modulator which in turns convert the sequence into electric signals so that we can transmit them onto the channel. The digital modulator maps the binary sequences into signal wave forms , for example if we represent 1 by sin x and 0 by cos x then we will transmit sin x for 1 and cos x for 0.

→ Digital modulation is used to transmit digital message signal by modulated carrier signal.

→The digital modulator maps the input binary sequernce of 1's & 0'S to analog signal wave forms.

→ ASK, FSK & PSK are examples of digital modulation techniques.

Digital modulation:

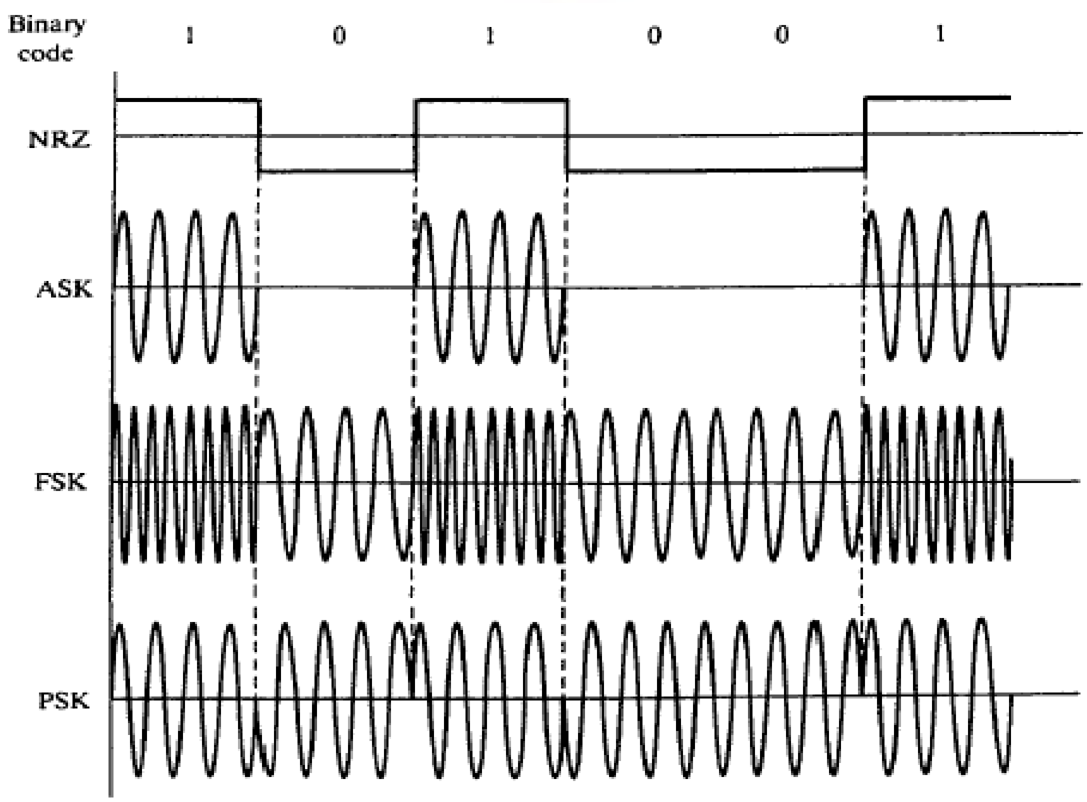


Fig. 5-16 Digital carrier modulation

Channel*:* The communication channel is the physical medium that is used for transmitting signals from transmitter to receiver. In wireless system, this channel consists of atmosphere , for traditional telephony this channel is wired , there are optical channels, under water acoustic channels etc.

Digital Demodulator*:* The digital demodulator processes the transmitted waveform which is corrupted by the channel and reduces the waveform to the sequence of numbers that represents estimates of the transmitted data symbols. So, here we have to demodulate the received signal that is coming from the channel . This block receives the analog signal and tries to estimate what value was transmitted here. So, if it is binary, it will try to estimate what is the bit that was transmitted here; from the wave form it receives. So, its output will be bits

Channel Decoder*:* Channel might have introduced some distortion in the analog signal due to which the digital demodulator detected some bits wrongly. So, if the digital demodulator detected some bits wrongly, the channel decoder will be able to correct those bits with the help of the extra information bits that were transmitted by the channel encoder.

Source Decoder *:*At the end, if an analog signal is desired then source decoder tries to decode the sequence from the knowledge of the encoding algorithm. And which results in the approximate replica of the input at the transmitter end.

Output Transducer*:* Finally we get the desired signal in desired format analog or digital. The point worth noting are :

1. The source coding algorithm plays important role in higher code rate

2. The channel encoder introduced redundancy in data

3. The modulation scheme plays important role in deciding the data rate and immunity of signal towards the errors introduced by the channel

4. Channel introduced many types of errors like multi path, errors due to thermal noise etc.

5. The demodulator and decoder should provide high BER.

<u>Disadvantages of digital communication over analog communication :</u>

→ Easy to generate the distorted signal.

→ Regenerative repeaters along the transmission path can deted a digital signal & retransmit a new clean (noise free) signal.

→These repeaters prevent accumulation of noise along the path.

→This is not possible with analog communication systems.

→The input to a digital system is in the form of a sequence of bits binary to M-array.

→ immune to distortion & interference.

→ Digital communication is rugged in the sense that it is more immune to channel noise & distortion.

→ Hardware is more flexible.

→Digital hardware implementation is flexible permits the use of microprocessor, mini processor digital switching & VLSI.

M – ary coding : $M = 2^K$

K → no. of bits

M: → no . of sysmbols.

<u>Case 1</u> : K = 1

M = 2

Symbols: 0

1

<u>Case 2:</u>

K = 2

$M = 2^2$

$= 4$

Symbols :	0	0
	0	1
	1	0
	1	1

Disadvantages of digital communication:

→ Although digital communication offers, so many advantages are discussed above, it has some drawbacks also. However, the advantages of digital communication system out weight disadvantages.

→Due to analog to digital conversion, the data rate becomes high. Therefore more bandwidth is required for digital signal transmission.

→Digital communication needs synchronization between transmitter & receiver.

Information rate: It is defined as the rate at which information transmitted per second.

It is also known as data transmission rate.

Example 4: in a digital telephone exchange voice signal is digitalized using 8- bits PCM. Calculate final bit rate of system.Voice: 300HZ – 3.4KH,BW = 3KHZ

Steps involved in analog to digital conversion

1) Sampling CT – DT

2) Quantization CA –DA

3) Encoding

$f_s \geq 2\ f_m$ samples / sec

Critical rate / nyquist rate: $f_s = 2\ f_m$

$$\Rightarrow f_s = 2 \times 3.4\text{ k} = 6800 \text{ samples / sec}$$

Practical rate: $f_s > 2\ f_m$

$$\Rightarrow f_s = 8000 \text{ samples / sec}$$

Sampling period: $T = 1/f_s$

$$\Rightarrow T = 1/8000 = 125\ \mu s \text{ samples / sec}$$

Bit rate : $R_s = n.\ f_s$ bits / sec

$$= 8 \text{ bits / sample} \times 8000 \text{ samples /sec}$$

$$= 64000 \text{ bits / sec}$$

SHANNON HARTLEY THEOREM & ITS IMPLICATIONS:

→ The capacity of a communication channel with band width "B" and additive Gaussian band limited white noise is

$$\boxed{C = B\log 2(1 + S/N)} \quad \text{bps} \rightarrow (1)$$

where C → capacity of communication channel.

B → band width

S → average signal power

N→ average noise power

$\frac{S}{N}$ → Signal to noise ratio

→equation (1) is referred to as " Shannon – Hartley " theorem.

→is of fundamental importance and has two important implications i.e., conclusions for communication system engineers.

FIRST IMPLICATION:

→calculation of upper limit or capacity of communication channel.

$C = B\log_2\left(1 + \frac{}{N}\right)$

Noise power spectral density

$$5 = \text{noise}\,\frac{\text{power}}{\text{bandwidth}} = \frac{N}{B}$$

$\Rightarrow N = 5B$

5 → Single sideband noise PSD

$\frac{y}{2}$→ double sideband PSD of white noise.

$C = B\log_2\left(1 + \frac{S}{yB}\right)$

$= B \times \frac{c}{y} \cdot \frac{y}{S} \log_2\left(1 + \frac{S}{yB}\right)$

$= \frac{yB}{S} \times \frac{S}{y} \log_2\left(1 + \frac{S}{yB}\right)$

$= \frac{S}{y} \log_2\left(1 + \frac{S}{yB}\right)^{\frac{yB}{S}}$

Let $x = \frac{c}{yB} \Rightarrow \frac{1}{s} = \frac{yB}{}$

B: bandwidth $\rightarrow \infty$

$x \rightarrow 0$

$\therefore\ C = \frac{c}{y} \log_2 (1 + x)^{1/s}$

$C = \frac{c}{y} \log_2 e$

$\therefore\ \boxed{C = 1.44 \frac{c}{y}}$ bps → (2)

→Eq (2) is referred as upper bound of capacity of communication channel.

<u>SECOND IMPLICATION :</u>

→second implication of Shannon Hartley theorem has to do with exchange of BW to signal to noise ratio.

Example 4: Suppose we want to transmit data at a rate of 10,000 bps over a communication channel having a BW – 3000 hz.

Procedure: To transmit data at a rate of 10,000 bps we need a communication channel with capacity 10,000 bps. If the channel capacity is less than the data rate then error less transmission is not possible.

→ **calculate signal to noise ratio required for the channel**

$C = B \log_2 (1 + S/N)$

$10{,}000 = 3000 \log_2 (1 + S/N)$

$\log_2 (1 + S/N) = \frac{10{,}000}{3000}$

$1 + S/N = 2^{3.33}$

$S/N = 10.05 - 1$

$S/N = 9.05$

→if we have a channel with bandwidth B = 10,000 HZ then we need

$\frac{C}{N} = 1$

→ Bandwidth reduction in 10,000 HZ to 3,000 hz results in an increase in signal to noise ratio from 1 to 9.

→ Shannon – Hartley theorem indicates a noise less channel has an infinite capacity, however when noise is present in the channel capacity does not approach infinity.

→because the noise power increases, as the communication channel bandwidth increases.

Example 5: A telephone channel has a BN of 3,000 hz and SNR 20 d B. Determine the channel capacity.

$SNR|_{dB} = 20dB$

$10 \log (S/N) = 20$

$S/N = 10^2 = 100$

$C = B\log_2 (1 + S/N)$

$= 19974$ bps ≅ 20 kbps

→ if SNR is increased to 25 dB. determine increased channel capacity.

$SNR|_{dB} = 25dB$

$10 \log (S/N) = 25$

$S/N = 10^{2.5} = 316.2$

$C = B\log_2 (1 + S/N) = 24925$ bps

≅ 25 kbps

Example 6: A tele communication channel has a BW 4 khz & SNR is 15. Calculate channel capacity.

$C = B\log_2 (1 + S/N)$

$= 4 \times 10^3 \log_2 (1+5) = 16$ kbps

→ Shannon Hartley theorem indicates that noise less channel has an infinite capacity.

→error less transmission is possible (Reliable data transmission) is possible if

$\boxed{R_b \leq C}$ → Channel coding theorem

Where R → bit rate / data rate / information rate / signaling rate

C → capacity of communication channel.

$\boxed{T = 1/R}$

Duration of pulse / bit duration

→ Bit rate is reciprocal of bit duration

→ Bit duration is reciprocal of bit rate.

Example 7: A memory less sources emits 8 messages which are equiprobable and source generates 100 symbols / sec. calculate information rate of source**.**

Source messages	source code words		
A	0	0	0
B	0	0	1
C	0	1	0
D	0	1	1
E	1	0	0
F	1	0	1
G	1	1	0
H	1	1	1

Average information of the source (H) = 3 bits / message

$\boxed{R_b = r\ H}$ → bits / messages

→ Messages / sec

= 3 b/m × 100 m/s

= 300 bps

Band rate = r_b / k →symbol rate.

PULSE MODULATIONS: Quite a few of the information bearing signals, such as speech, music, video, etc., are *analog* in nature; that is, they are functions of the continuous variable *t* and for any *t* = *t*1, their value can lie anywhere in the interval, say − *A* to *A*. Also, these signals are of the baseband variety. If there is a channel that can support baseband transmission, we can easily set up a baseband communication system. In such a system, the transmitter could be as simple as just a power amplifier so that the signal that is transmitted could be received at the destination with some minimum power level, even after being subject to attenuation during propagation on the channel. In such a situation, even the receiver could have a very simple structure; an appropriate filter (to eliminate the out of band spectral components) followed by an amplifier. If a baseband channel is not available but have access to a passband channel, (such as ionospheric channel, satellite channel etc.) an appropriate CW modulation scheme discussed earlier could be used to shift the baseband spectrum to the passband of the given channel. Interesting enough, it is possible to transmit the analog information in a digital format.

Though there are many ways of doing it, in this chapter, we shall explore three such techniques, which have found widespread acceptance. These are: Pulse Code Modulation (PCM), Differential Pulse Code Modulation (DPCM)and Delta Modulation (DM). Before we get into the details of these techniques, let us summarize the benefits of digital transmission. For simplicity, we shall assume that information is being transmitted by a sequence of binary pulses.

i) During the course of propagation on the channel, a transmitted pulse becomes gradually distorted due to the non- ideal transmission characteristic of the channel. Also, various unwanted signals (usually termed interference and noise) will cause further deterioration of the information bearing pulse. However, as there are only two types of signals that are being transmitted, it is possible for us to identify (with a very high probability) a given transmitted pulse at some appropriate intermediate point on the channel and *regenerate* a clean pulse. In this way, be completely eliminating the effect of distortion and noise till the point of regeneration. (In long-haul PCM telephony, regeneration is done every few Kilometers, with the help of *regenerative repeaters.*) Clearly, such an operation is not possible if the transmitted signal was analog because there is nothing like a reference waveform that can be regenerated.

ii) *Storing* the messages in digital form and forwarding or redirecting them at a later point in time is quite simple.

iii) *Coding* the message sequence to take care of the channel noise, encrypting for secure communication can easily be accomplished in the digital domain.

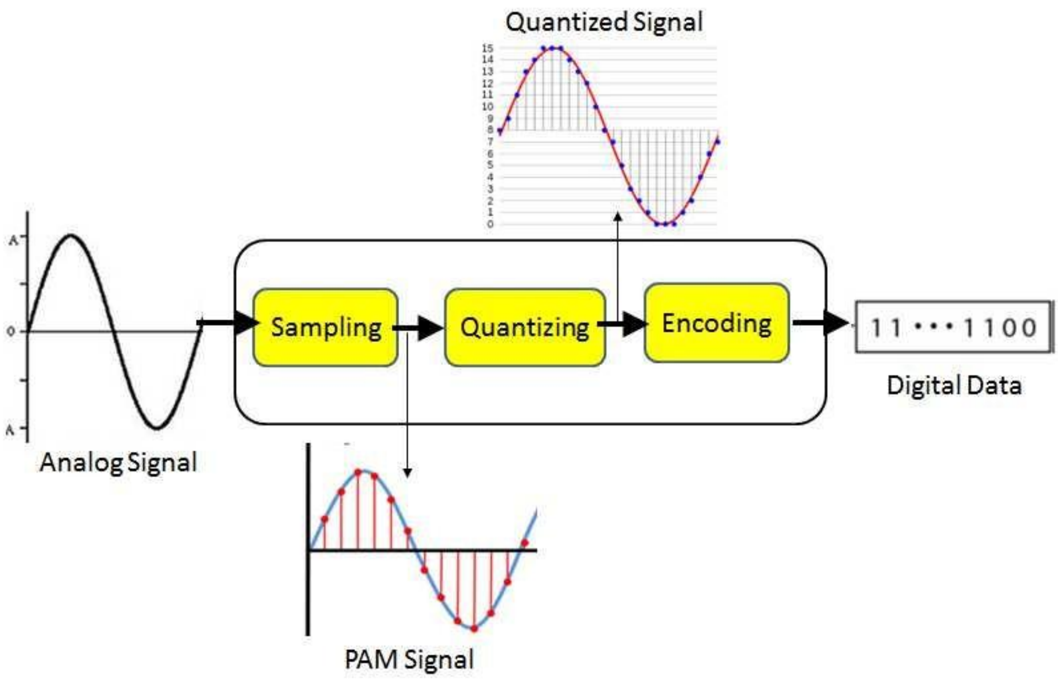
iv) *Mixing* the signals is easy. All signals look alike after conversion to digital form independent of the source (or language!). Hence they can easily be multiplexed (and demultiplexed)

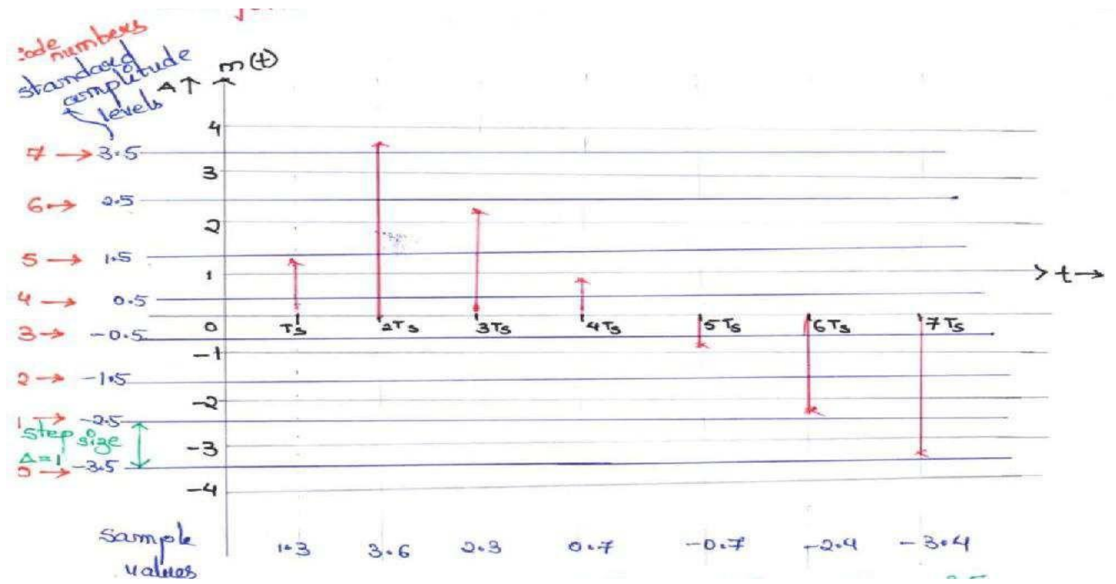
Pulse modulation

Pulse analog modulation	pulse digital modulation
1) PAM	1) PCM
2) PWM	2) differential PCM
3) PPM	3) Delta modulation
	4) Adaptive delta modulation

The PCM system: It is defined as a modulation process in which analog message signal is represented by sequence of code d pulses (sequence of pulses).Two basic operations in the conversion of analog signal into the digital is *time discretization* and *amplitude discretization*. In the context of PCM, the former is accomplished with the *sampling operation* and the latter by means of *quantization*. In addition, PCM involves another step, namely, conversion of quantized amplitudes into a sequence of simpler pulse patterns (usually binary), generally called as *code words*. (The word *code* in pulse code modulation refersto the fact that every quantized sample is converted to an *R* -bit code word..

The following figure shown generation of PCM signal.





Quantization level	1.5	3.5	2.5	0.5	-0.5	-2.5	-3.5
Code numbers	5	7	6	4	3	1	0
	101	111	110	100	011	001	000 →Binary representation

QUANTIZER: It is defined as the process of converting discrete time continuous amplitude signal into discrete time discrete amplitude signal.

→ Quantization is a approximation process The quantizer converts each sample to one of the values that is closest to it from among a pre-selected set of discrete amplitudes. The encoder represents each one of these quantized samples by an R -bit code word. This bit stream travels on the channel and reaches the receiving end.

→ to perform quantization we consider standard amplitude levels (representation levels) which are finite

→ let us denote standard amplitude levels with

$$L = 2^n \text{ (representation levels)}$$

Where n → no. of binary digits (bits) transmitted per sample.

→ let us consider n = 3

$L = 2^n = 2^3 = 8$ levels

→ Quantization is a process of rounding off the sample values to their nearest quantization levels (standard representation levels / reference levels).

Step size = R / L

⇒in this examples $\Delta = 8/8 = 1$

→ Quantized sample value

$T_s \rightarrow 1.5$; $2T_s \rightarrow 3.5$; $3T_s \rightarrow 2.5$; $4T_s \rightarrow 0.5$; $5T_s \rightarrow 0.5$; $6T_s \rightarrow 2.5$; $7T_s \rightarrow 3.5$

→ Code numbers

$T_s \rightarrow 5$; $2T_s \rightarrow 7$; $3T_s \rightarrow 6$; $4T_s \rightarrow 4$; $5T_s \rightarrow 3$; $6T_s \rightarrow 1$; $7T_s \rightarrow 0$;

→ Binary representation

$T_s \rightarrow 101$; $2T_s \rightarrow 111$; $3T_s \rightarrow 110$; $4T_s \rightarrow 100$; $5T_s \rightarrow 011$; $6T_s \rightarrow 001$; $7T_s \rightarrow 000$;

→ o/p digital signal generated at

(Bit rate) – R_b = No. of samples / sec no. of bits / samples = $n \times f_s$

$= 7 \times 3$

= 21 bps

Example 8: A TV signal of BW 4.2 MHZ is transmitted using binary PCM, with the number of representation level of 512. Calculate the following

1) Code word length.

2) Final bit rate

3) calculate transmission BW.

Given : BW = 4.2 MHz

$L = 512 = 2^9$

$\Rightarrow n = 9$ bits

$f_m = 4.2$ MHZ

$f_s \geq 2f_m$

$\geq 2(4.2)$ MHz

$\geq$ 8.4 MHz

→ code word length = 9 bits (PCM word / no. of bits transmitted per sample)

→ Bit rate = $f_s \times n$

$R_b = 9 \times 8.4$ Mbps

= 75.6 Mbps.

→bandwidth should be at least 75 Mbps $B_T = \frac{1}{2} R_b$ = 38 Mbps

Example 9: A PCM system uses a uniform quantize followed by a 7 bit binary encoder. The bit rate of the system is 50×10^6 bps. What is the minimum message BW

Given n = 7

R_b = 50 Mbps

$R_b = n \times f_s$

50 Mbs = $7 \times f_s$

$\Rightarrow f_s$ = 7.14Mhz

BW = f_m = 3.5 MHZ

$\therefore f_S = 2 f_m$

LINE CODING FOR DIGITAL SIGNAL TRANSMISSION:

→ Line coding is defined as electrical representation of binary digits.

→ the best way to map bits/ symbols into pulse wave form is line coding.

→ Line coding is used for transmission of binary data.

P → For the message sequence 110 101 0010 , draw the

1) unipolar NRZ wave form

2) unipolar RZ waveform

3) polar NRZ waveform

4) polar RZ waveform

5) Bipolar / AMI (Alternate Mask inversion)

R_b = 10 bps

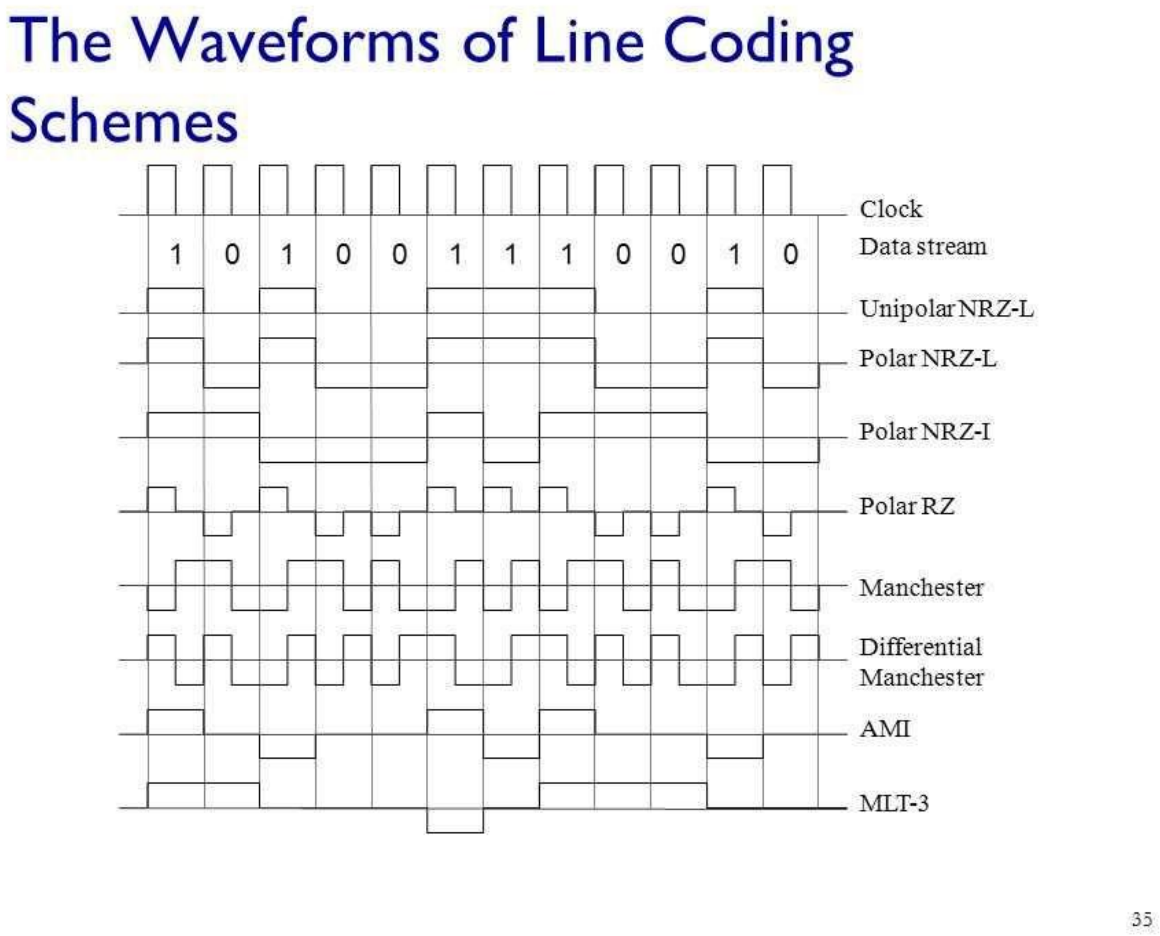
Duration of each pulse

$\boxed{T_b = 1/R_b}$

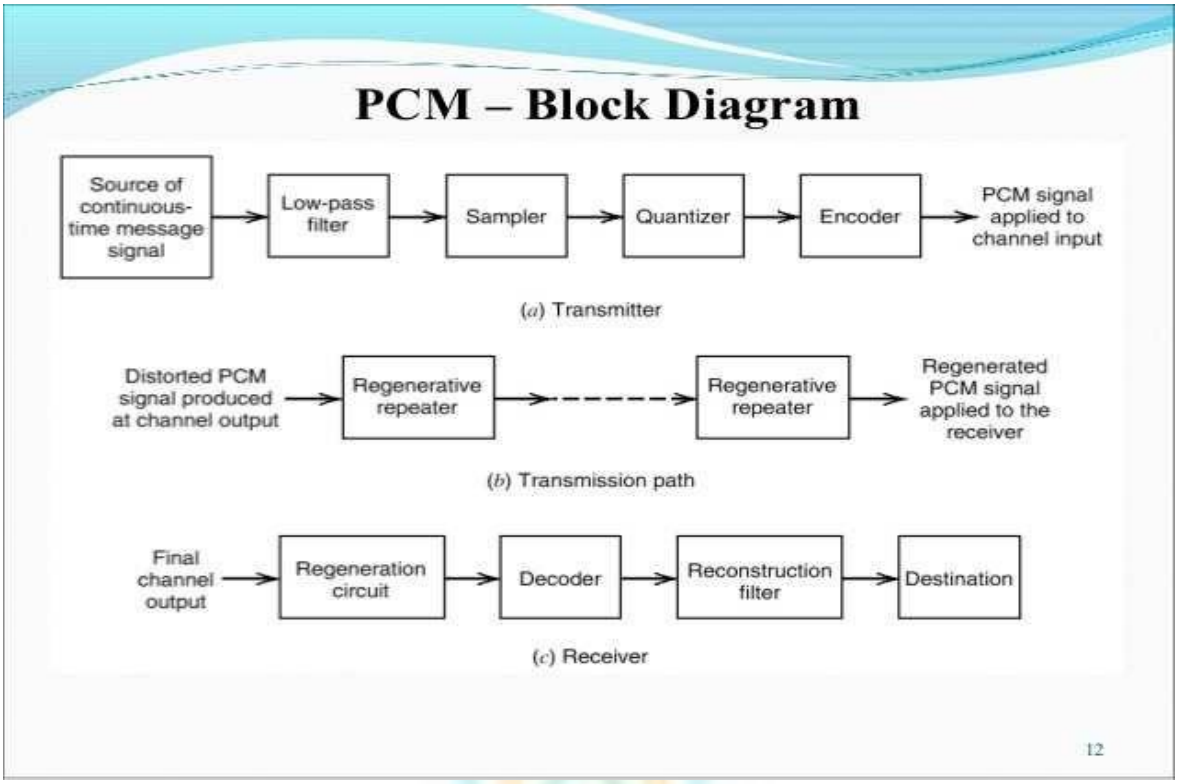
= 1/10 = 0.1 sec / bit

→Reciprocal of bits rate is bit duration

→Reciprocal of bit duration is bit rate



PCM system: generation & Reconstructions: A PCM communication system is shown in the fig below.The analog signal is sampled and then the samples are quantized and encoded.The output of the encoder is a binary sequence. The combination of the quantizer and encoder is often called as *analog to digital(A/D)converter.* The digitally encoded signals is transmitted over the communication channel to the receiver.The reconstructed signal is identical with the input except for the quantization noise and another noise component that results from decoding errors due to channel noise.



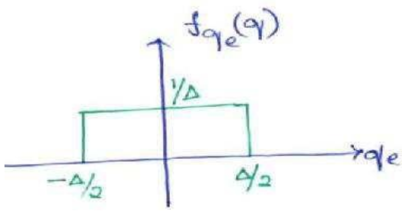
CALCULATION OF QUANTIZATION NOISE POWER:

$-\frac{\Delta}{2} \le qe \le \frac{\Delta}{2}$

→Assume that "qe" quantization error in a uniform random variable

$F_{qe}(qe) = \frac{\Delta}{2}$; $-\frac{\Delta}{2} \le qe \le \frac{\Delta}{2}$

= 0 ; otherwise



Uniform pdf

→ mean value of quantization error

$E[E] = E = \int_{-\infty}^{\infty} E\, f_E(E)\, dE$

$$= \int_{-\Delta/2}^{\Delta/2} E.\frac{1}{\Delta}De$$

$$= \frac{1}{\Delta}\ \frac{\epsilon^2}{2}\Big|_{-\Delta/2}^{\Delta/2}$$

$$= 0$$

→ mean square value

$$E[e^2] = \int_{-\infty}^{\infty} e^2 f_e(\epsilon)\, d\epsilon = \int_{-\Delta/2}^{\Delta/2} \epsilon^2 \cdot {}^{1}/_{\Delta}\, d\epsilon$$

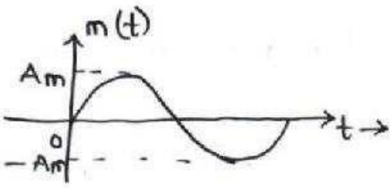
$$= \frac{1}{\Delta}\left(\frac{e^3}{3}\right)_{-\Delta/2}^{\Delta/2}$$

$$\boxed{E[e^2] = \Delta^2/12}$$

→Quantization error introduces, distortion in the desired signal

→ there it is also known as quantization noise / quantization distortion.

Ex:



mean square value v^2 RMS $= \frac{A2m}{2}$ → power

→ mean square value of quantization noise is $\frac{\Delta^2}{12}$

(power of quantization noise)

→power of quantization noise depends on step size.

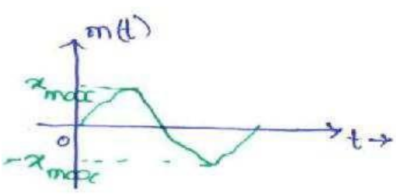
→ to reduce quantization noise power reduce the step size

Step size $\boxed{\downarrow \Delta = \frac{R}{L\uparrow} = \frac{R}{2^{ng}}}$

→ in PCM quantization error is reduced at the cost of increase in BIT RATE.

Calculation of signal power:

→ consider i/p to the PCM is a random message signal.



Range R = $x_{max} - (-x_{max})$

$\boxed{R = 2x_{max}}$

Where R=Range of input signal

→ Signal power (S) = p Watts P ≤ 1 Watt → (1)

Normalized signal power (S) = 1 Watt

S2: quantization noise power

$$\boxed{N_q = \frac{\Delta^2}{12}} \rightarrow (2)$$

S3: calculate signal to quantization noise power

$$\Longrightarrow \frac{S}{N_q} = \frac{P}{\frac{\Delta^2}{12}}$$

$$= \frac{\text{NormaSSized cignaS power}}{\text{Quantization noice power}}$$

$$= \frac{S}{N_q} = \frac{P}{\frac{\Delta^2}{12}} = \frac{12\,p}{\Delta^2}$$

$$= \frac{12\,p}{\Delta^2}$$

$$= \frac{12\,p}{(R/L)^2}$$

$$= \frac{12p\,.\,L^2}{R^2}$$

$$= \frac{12p\,.2^{2n}}{4s^2_{max}}$$

$$= \frac{3p4^n}{s^2_{max}}$$

P = 1 Watt

X_{max} = 1V

$$\boxed{\frac{Æ}{N_q} = 3.2^{2n}} \rightarrow (3)$$

Where n → no . of bits / Sample

$SNR_q = 3.2^{2n}$

SNR_q / in dB = 10 $\log_{10} 3.\ 2^{2n}$

= 10 $\log_{10} 3 + 10 \log 3.2^{2n}$

$= 4.8 + 2$ on $\log_{10} 2$

$\uparrow \boxed{SNR_q \big|_{dB} = 4.8 + 6\,n \uparrow}$ → i/p to the PCM is random signal.

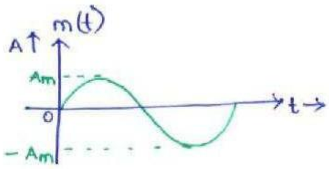
→ to maximize SNR_q , increase no. of bits / sample.

<u>DERIVATION FOR SIGNAL TO QUANTIZATION NOISE RATIO IN PCM</u> :(i/p to PCM is single tone sinusoidal message signal):

S1) calculate signal power

$$S = \frac{Æm^2}{2}$$

$R = 1\ \Omega$



Normalized signal power

$$\boxed{S = \frac{Æm^2}{2}} \rightarrow (1)$$

S2) Quantization noise power

$$\boxed{N_q = \frac{\Delta^2}{12}} \rightarrow \quad (2)$$

S3) Calculate signal to Quantization noise power ratio

$$\frac{S}{N_q} = \frac{\frac{Am^2}{2}}{\frac{\Delta^2}{12}} \rightarrow \quad (3)$$

$$= \frac{6Æm^2}{\frac{R^2}{L^2}}$$

$$= \frac{6Æm^2 . 2^{2n}}{4Æm^2}$$

$$\boxed{SNR_q = 1.5 \times 2^{2n}} \quad \rightarrow (4)$$

$$SNR_q \big| dB = 10 \log_{10}(1.5 \times 2^{2n})$$

$$= 10 \log_{10}(1.5\ ) \times 20n \log_2(2)$$

$$= 1.76 + 6n$$

$$\boxed{SNR_q \big|_{dB} = 1.8 + 6\,n \uparrow}$$

For every 1 bit increase in "n" increases the signal to quantization noise ratio by 3 dB.

→ This rule in PCM is known as " 6 dB rule in PCM"

Example 10: In a PCM system each quantization level is encoded into 8 bits. The SNR_q =

a) $\frac{1}{12}(1/256)^2$ b) 48 dB   c) 64 dB   d) 256 dB

$SNR_q = 1.8 + 6\,n$

$= 1.8 + 6(8)$

$\cong$ 48 dB

Example 11: Information in an analog wave form with a max freq fm = 3khz, is to be transmitted using PCM. The quantization distortion is specified not to exceed $\pm$ 1% of peak analog signal.

a) what is minimum require sampling rate.

b) what is the min no. of bits/ sample (or) bits/ PCM word that should be used in digitizing the analog wave form.

c) what is resulting R_b

d) what is transmission BW

Given fm = 3 KHz

$\text{c} \leq \pm 1\% \, V_p$

a) Minimum sampling rate

fs = 2fm

= 2(3) KHz

= 6000 Samples / sec.

b) Minimum no. of bits / Sample

$R = V_p$

$L = 2^n$

$\text{c} = \pm 1\% \, V_p$

$0.09\, V_p \leq \text{c} \; 0.01\, V_p$

$\text{c} = \frac{\Delta}{2} \leq 0.01\, V_p$

$\frac{R}{L} \leq 0.02\, V_p$

$\frac{V_p}{2^n} \leq 0.02\, V_p$

$\Rightarrow n = 5.6 \cong 6$

c) Resulting bit rate:

$R_p = n \times f_s = 6 \times 6000$

$= 36{,}000$ bps.

d) Transmission BW

$C \geq 36{,}000$ bps.

$\Rightarrow BW = \frac{1}{2} R_p = 18{,}000$ bps

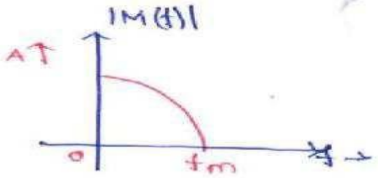
Min transmission BW required for transmission of PCM signal is

$B_T = \frac{1}{2} R_b$

$= \frac{1}{2} n \times f_s$

$= n \times f_m$

→ Figure shows spectrum of analog message signal.



→ In pulse code modulation system quantization distortion (qe) is reduced at the cost increases in bit rate. Which intern effects the " Transmission Bandwidth"

MODIFIED VERSIONS OF PCM:

1) DPCM : differential pulse code modulation.

2) ADPCM: Adaptive differential pulse code modulation

3) Delta modulation (DM) / 1 Bit PCM.

(Δ - Modulation)

4) ADM: Adaptive Delta modulation

p→ Find the Fourier transform of following rectangular pulse and sketch the spectrum of it.

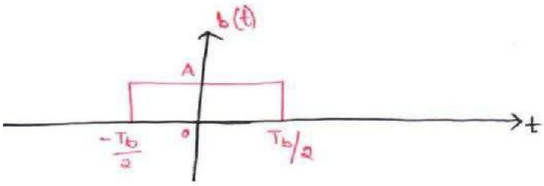


Fig: shows rectangular pulse

F { b(f) } = b(f) = $f_{-\infty}^{\infty} b(t).e^{-2njft}$ dt

$= f_{-Tb/2}^{Tb/2} Ae^{-j2nft}$ dt

$= \frac{Æ}{-j2nf} [\, e^{-j2nft} \,]_{-Tb/2}^{Tb/2}$

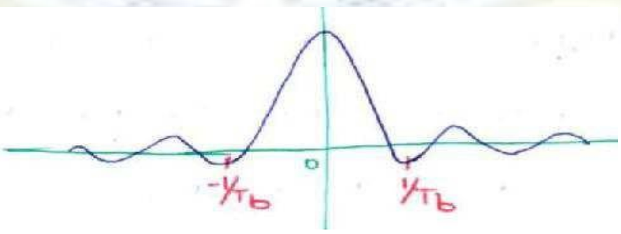
$= \frac{Æ}{-j2nf} [e^{-jnfT_b} - e^{jnfT_b}]$

$= \frac{ÆT_b}{nfT_b} \left[\frac{e^{jgfT_b} - e^{-jgfT_b}}{2\,j} \right]$

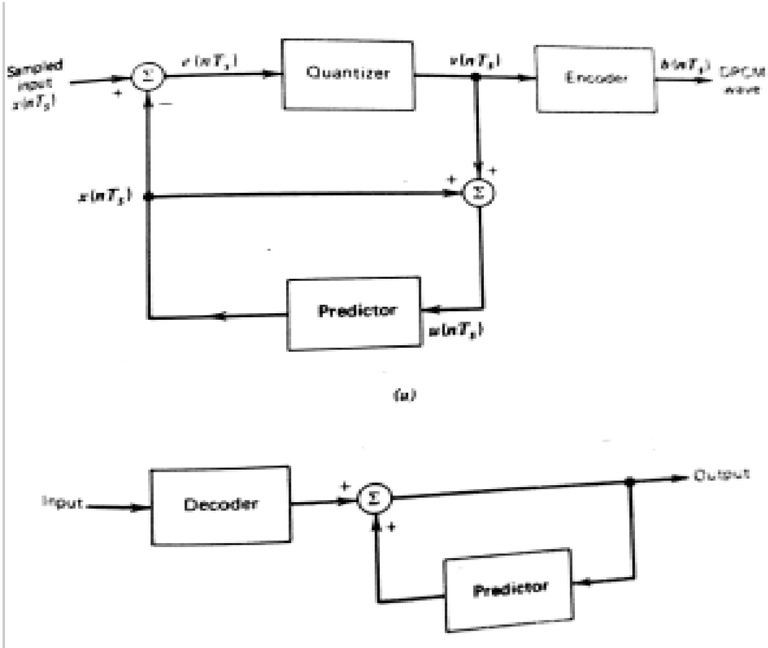
$= \frac{ÆT_b}{nfT_b} \sin(nfT_b)$

$= AT_b \sin \frac{(nfT_b)}{(nfT_b)}$

b(f) = $AT_b \sin c\,(fT_b)$



DIFFERENTIAL PULSE CODE MODULATION: If the difference in the amplitude levels of two successive samples is transmitted rather than the absolute value of the actual sample is called differential PCM (DPCM)



DPCM Encoder and decoder

$$\rightarrow e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

Adaptive Differential Pulse-Code Modulation (ADPCM):

Need for coding speech at low bit rates , we have two aims in mind:

1. Remove redundancies from the speech signal as far as possible.
2. Assign the available bits in a perceptually efficient manner.

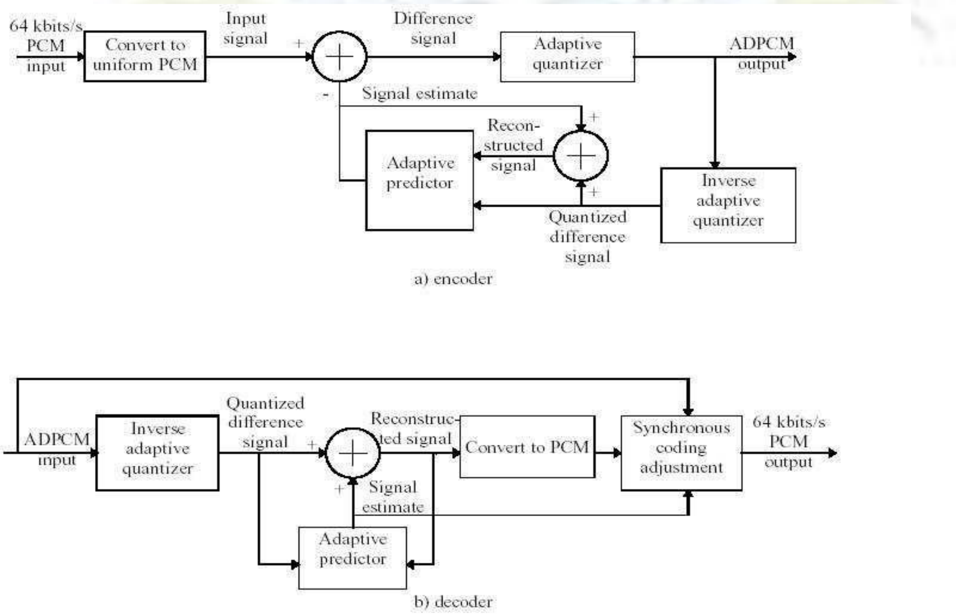


Figure 1: ADPCM Encoder and Decoder

DIFFERENCE BETWEEN ACTUAL SAMPLE AND PREDICTED SAMPLE.

→ in PCM, each sample is encoded with “n” bits where as in DPCM, difference between actual sample and its predicted sample value is encoded with “n” bits.

→$e(nT_s)$ range is small when it is compared to range of actual sampled signal. Therefore we need lesser number of quantization levels to perform quantization.

→ Bandwidth required for DPCM signal is less when it is compared to the PCM.

→DPCM is used in (cellular , telephony) personal communication services to digitize voice message signal.

→DPCM predictor (delay line filter) predicts future sample values.

→ Predicted sample value may not be exactly equals to actual sample value. But it is very close to actual sample value.

→ In DPCM quantization error is reduced with number of quantization levels i.e. less number of bits.

n → less

L →2^n → less

→ q_e → less

→ Bitrates decreases R_b → less

$\Longrightarrow BW_T \rightarrow$ less

<u>DELTA MODULATION / 1 BIT PCM:</u> Delta modulation(DM) uses a single-bit DPCM code to achieve digital transmission of analog signal. Delta modulation was introduced in the 1940s as a simplified form of pulse code modulation (PCM), which required a difficult-to-implement analog-to-digital (A/D) converter. The output of a delta modulator is a bit stream of samples, at a relatively high rate (eg, 100 kbit/s or more for a speech bandwidth of 4 kHz) the value of each bit being determined according as to whether the input message sample amplitude has increased or decreased relative to the previous sample. It is an example of differential pulse code modulation (DPCM).

Principle: In delta modulation, 1 bit is transmitted for each i/p sample (each i/p sample is encode with 1bit).

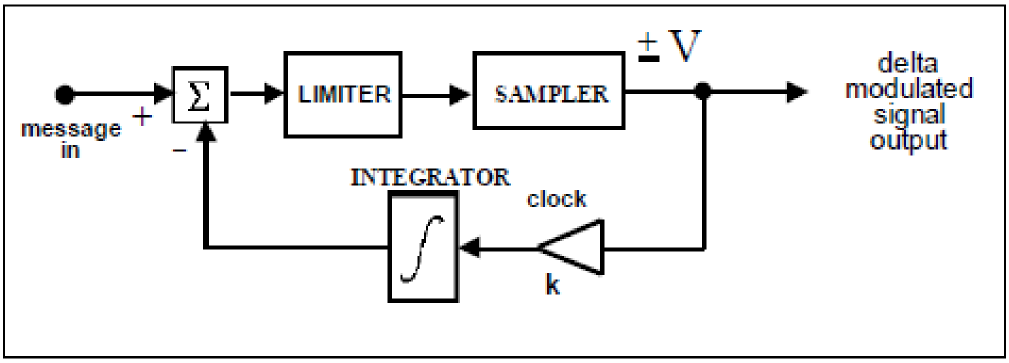
→ Delta modulation makes use of two level quantize.

Quantization levels.	Bit transmitted.
+ Δ	1
- Δ	0

→ input and output characteristics of 2 – level quantize

Block diagram

The operation of a delta modulator is to periodically sample the input message, to make a comparison of the current sample with that preceding it, and to output a single bit which indicates the sign of the difference between the two samples. This in principle would require a sample-and-hold type circuit.



Advantages of Delta modulation

1. DM transmits only one bits per sample, thus the signaling rate and the transmission channel bandwidth is quite small.

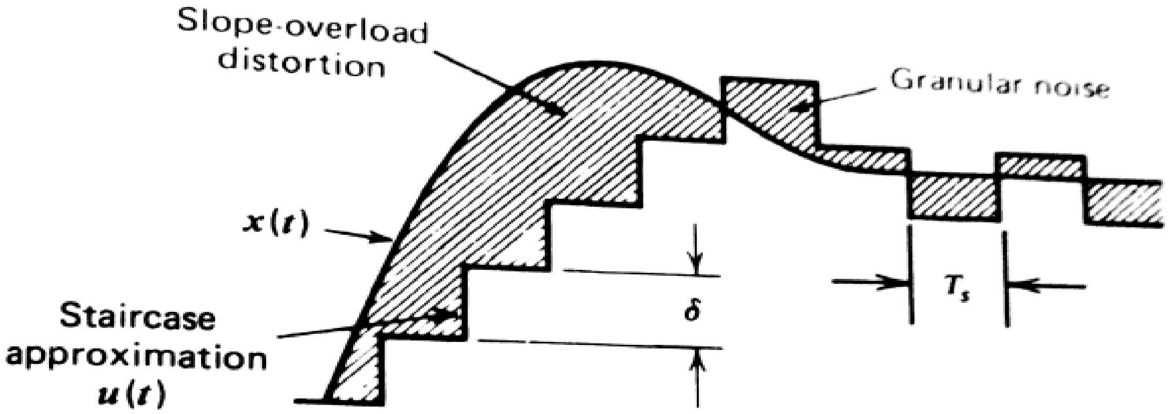
2. The transmitter and receiver implementation is very simple for delta modulation.

There are two limitations for delta modulation:

1 .Slope overload distortion

2 .Grannular noise

1. Slope Overload distortion: It is occurred when the approximated signal m^(t) does not follow the signal m(t).In this case the original baseband signal cannot be recovered without any distortion.



Slope Overload anf Grannular noise

In slope overload distortion ,slope of the signal m(t) is more positive or more negative than the slope of m^(t).The rate of rise input signal m(t) is so high that the staircase signal cannot approximate it.The step size S becomes too small for staircase signal m^(t) to follow the steep segment of m(t).Thus there is a large error between the staircase approximated signal and original input signal m(t).To reduce this error the stepsizeshould be increased when slope of the sinal is high.

2. Grannular Noise: This occurs when the step size is too large compared to small variations in the input signal.For every small variations in the input,the staircase signal is changed by a large amount because of large step size.The solution to these problem is to make the step size small.Thus large step size is required to accommodate wide range of input signal to reduce slope overload distortion and small step size is required to reduce grannular noise.Adaptive delta modulation is uded to overcome these errors

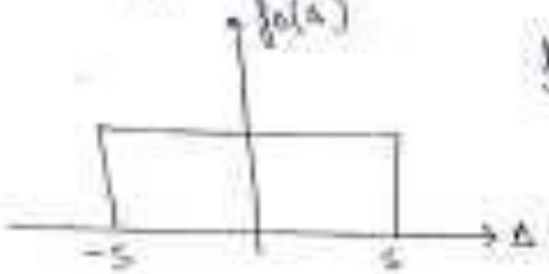
→ Granular distortion can be decreased by decreasingΔ.

→ $\boxed{B_T \text{ of PCM} > B_T \text{ of DPCM} > B_T \text{ of DM}}$

Quantization Noise in Delta Modulation :

$$\varepsilon_{rms} = m(t) - \hat{m}(t) = \Delta(t)$$

Since $\Delta(t)$ varies randomly δ is assumed to be an uniformly distributed random function whose PDF is given by $f_\Delta(\Delta)$



$$f_\Delta(\Delta) = \begin{cases} 0, & \Delta < -S \\ \frac{1}{2S}, & -S < \Delta < S \\ 0, & \Delta > S \end{cases}$$

$$(N_q)_{DM} = \int \Delta^2 \cdot f_\Delta(\Delta) \cdot d\Delta = \int_{-S}^{S} \Delta^2 \cdot \frac{1}{2S}\, d\Delta = \frac{1}{2S}\int_{-S}^{S} \Delta^2\, d\Delta$$

$$= \frac{1}{2S}\left[\frac{\Delta^3}{3}\right] = \frac{1}{2S}\left[\frac{S^3}{3} + \frac{S^3}{3}\right] = \frac{1}{2S} \cdot \frac{2S^3}{3} = \frac{S^2}{3}$$

$$(N_q)_{DM} = \frac{S^2}{3}$$

$$[SNR]_q \text{ in DM} = \frac{S_0}{N_q}$$

$$S_0 = S_i = \overline{m^2(t)}$$

Let $m(t) = A\cos\omega_m t$

$$\overline{m^2(t)} = \frac{A^2}{2}, \quad A = \frac{S}{2\pi f_m T_s}$$

$$(SNR)_q \text{ in DM} = \frac{\left(S/2\pi f_m T_s\right)^2 / 2}{S^2/3}$$

Let for DM, the bit rate $= 1/\tau$ where τ = Duration of each bit

ADM Reciever

UNIT - II
INFORMATION THEORY

→Let us consider a message " tomorrow onwards sun rises in east" = m_k
P_K → probability of occurrence of message.
I_k → Information content in message m_k
$I_k \propto \frac{1}{p_k}$
$I_k = k.\frac{1}{p_k}$
$I_k = \log_2(1/p_k)$
→ This is an logarithmic measure of information content in a message "m_k"
→ Unit for information – Bit, i
→ If base of logarithm is 2.
If base of logarithm is 10 → Unit Hartley (OR) Decit
If base of logarithm is 'e' them unit → "nat"

EX:
1) A Source produces one of four possible symbols during each interval having probabilities.
$P(x_1) = ½; P(x_2) = ¼ ; P(x_3) = 1/8;$
Obtain the information content of each of these symbols
$I(x_i) = \log_2 (1/(x_i))$ bits
$I(x_1) = \log_2 (1/1/2)$ = 1bits information content
$I(x_2) = \log_2 (1/1/4)$ = 2bits in a symbol / message x_i
$I(x_3) = \log_2 (1/1/8)$ = 3bits = $I(x_4)$

PROPERTIES OF INFORMATION:
1) $I(x_i) = 0$, if $p(x_i) = 1$ → Int. content in x_i is 'o' if is certain event.
2) $I(x_i) \geq 0$ for $1 \geq p(x_i) \geq 0$
3) $I(x_i) > I(x_j)$ for $p(x_j) > p(x_i)$

→ What is entropy? Derive its expression. Define information rate. Write down the derivation for average information "H" for the case of two message. P & 1-P and also out the maximum value 'H'

ENTROPY:
→ Entropy is defined as average information of a source, and denoted by 'H'
Unit : bits / message

DERIVATION FOR EXPRESSION FOR ENTROPY OF A SOURCE (H):
→ Consider a DMS (discrete memory less source) producing messages $m_1, m_2. . . . m_m$ with the probabilities $p_1, p_2. . . P_m$ respectively.
→ In a longer time interval source emitting " L" messages (L >> M)
→ In 'L' messages , message m_1 will appear P_1L times
Message m_2 will appear P_2L times
:
:
:
Message m_m will appear $P_M L$ times
→ $I(m_1) = \text{Log}_2(1/P_1)$ bits
:
:
→ $I(m_M) = \text{Log}_2(1/P_M)$ bits
→ $I_{total} = P_1^L I(m_1) + P_2^L I(m_2) + \ldots\ldots.+ P_M^L\, I (m_M)$
Total information content in 'L' messages
→ Average information of a source

$$H = \frac{I_{total}}{L} \text{ bits / messages.}$$

$$= \frac{P_1^L I(m_1) + P_2^L I(m_2) + \cdots + P_M^L I(m_M)}{L}$$

$$H = \sum_{i=1}^{M} P_i I(m_i)$$

$$H = \sum_{i=1}^{M} log_2 \left(\frac{1}{p_i}\right)$$

$$H = \sum_{i=1}^{M} log_2 \left(\frac{1}{p_i}\right)^{P_i}$$

→ Two messages with probabilities p & 1-P

$H = p\log_2 \frac{1}{2} + (1\text{-}p) \log_2 \left(\frac{1}{1} - p\right)$

→ Entropy if a source 'x'

$H(x) = \sum_{i=1}^{M} p(xi) \, log_2 \frac{1}{p(xi)}$ bits/symbol

$= -\sum_{i=1}^{M} p(xi) \, log_2 \, p(xi)$ bits/symbol

<u>DERIVATION FOR "ENTROPY" OF A SOURCE FOR THE CASE OF TWO MESSAGES P & 1-P.</u>

$H = \sum_{k=1}^{M} P_k \, log_2 \frac{1}{p_k}$ bits / symbol

$= P_1 \log_2 \frac{1}{p_1} + P_2 \log_2 \frac{1}{p_2}$

$= P_1 \log_2 \frac{1}{p} + (1\text{-}P) \log_2 \frac{1}{(1-p)}$ bits / message.

<u>CONSIDER FOR H_{MAX} (MZX. VALUE OF ENTROPY) :</u>

$$\frac{d\,H_{max}}{dp} = 0$$

$$\frac{d}{dp}\left(P \ \log_2 \frac{1}{p}\right) + \ (1-p)\log_2 1/(1-p)$$

$$\log_2 \frac{1}{p} + P \frac{d}{dp} \log_2 \left(\frac{1}{p}\right) + \{-\log_2\left(\frac{1}{1-p}\right)\} + (1\text{-}p).\ \frac{d}{dp} \log_2 \left(\frac{1}{1-p}\right) \ = 0$$

$$\log_2 \frac{1}{p} + P \frac{1}{\frac{1}{p}} \left(\frac{1}{p^2}\right) - \log_2 \left(\frac{1}{1-p}\right) + \ (1-p) \frac{1}{\frac{1}{(1-p)}} \frac{(-1)}{(1-p)^2} \ = 0$$

$$\log_2 \frac{1}{p} - 1 - \log_2 \left(\frac{1}{1-p}\right) + 1 = 0$$

$\text{Log}_2 \frac{1}{p} = \log_2 \left(\frac{1}{1-p}\right)$

$\Rightarrow \frac{1}{p} = \frac{1}{1-p}$

1-p = p

2p = 1

$P = \frac{1}{2}$

$\frac{dH}{dp} = \log_2 \left(\frac{1-p}{p}\right) = 0$

$\frac{1-p}{p} = 2^0$

1-p = p

2p = 1

$P = \frac{1}{2}$

→ Maximum entropy of a source is obtained, if a source emits messages which are equiprobable.

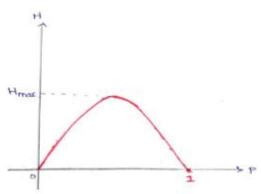


Figure shows H v/s P.

<u>INFORMATION RATE OF A SOURCE:</u>

→ Information rate of a source is referred as the rate at which information transferred per second and it is denoted. By "R"

$$\boxed{R = H\,R_S}$$

R_S → symbols / sec

H → bits / symbol

R → bits/sec

$\boxed{R = rH}$ Where r → symbol / message rate

PROPERTIES OF ENTROPY: H(X)

→ Average information of a source 'x'

1) $H(x) \geq 0$: Average information of source is always non negative.

2) $0 \leq H(x) \leq H_{max} = \log_2^M$

Where M – no .of messages generated by source, which are equiprobable.

3) Maximum value of entropy of a source is given by

$$H(x) = \log_2^M \text{ bits / message}$$

Given a telegraph source having two symbols ' . ' & '-' the probability of dots occurring is twice that of dash calculate H.

$p(\cdot) = 2\, p(-1)$

$x_1 \quad x_2$

$p(-) = 1/3 = P_1$

x_2

$p(\cdot) = 2/3\ P_2$

x_1

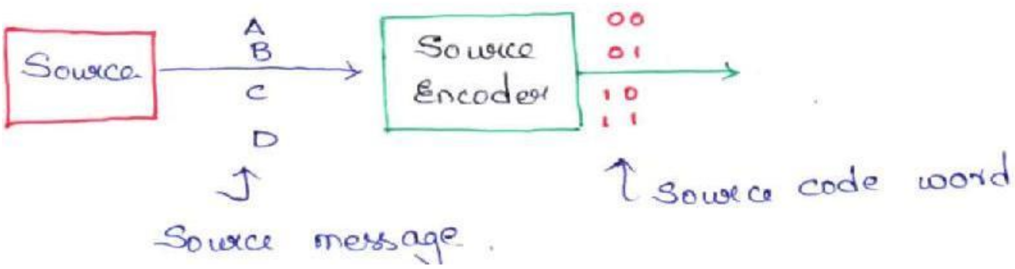
$H = P_1 \log_2 \frac{1}{p_1} + P_2 \log_2 \frac{1}{p_2}$

$H = \frac{1}{3}\log_2 \frac{1}{\frac{1}{3}} + \frac{2}{3}\log_2 \frac{1}{\frac{2}{3}}$

H = 0.918 bits/ message

$\boxed{\therefore H = 3.36 \sum_{K=1}^{M} P_k \log_{10} 1/p_k}$

SOURCE ENCODING



→ For each source encoding technique calculate the following parameters

1) Efficiency

$\boxed{\eta = \frac{H}{L} \times 100}$ $\bar{L} = \sum_{K=1}^{M} P_k L_k$

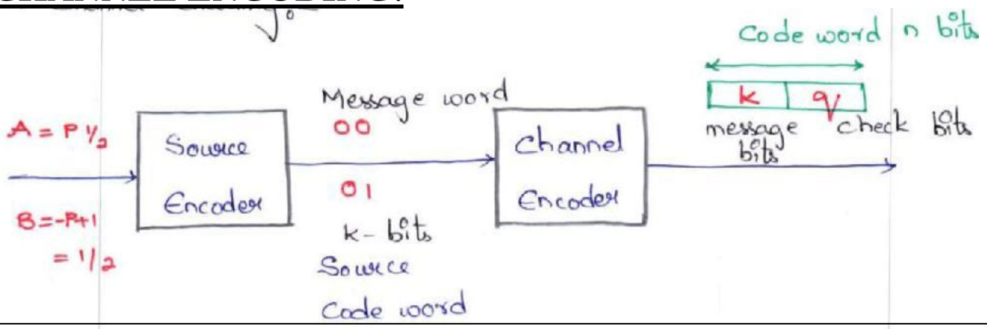
Whe

$\bar{L}$ → Average length of code word.

2) REDUNDANCY:

Redundancy = $1 - \eta = 1 - \frac{H}{L}$

CHANNEL ENCODING:



$\therefore N = k + q$

$q = n - k$

CHANNEL ENCODING TECHNIQUES:

1) Linear block codes
2) cyclic code
3) Convolution codes.

A DMS emits 8 symbols x_i i = 1,2,3,. . . 8 which are equally likely. Construct (Develop) source codes words using.

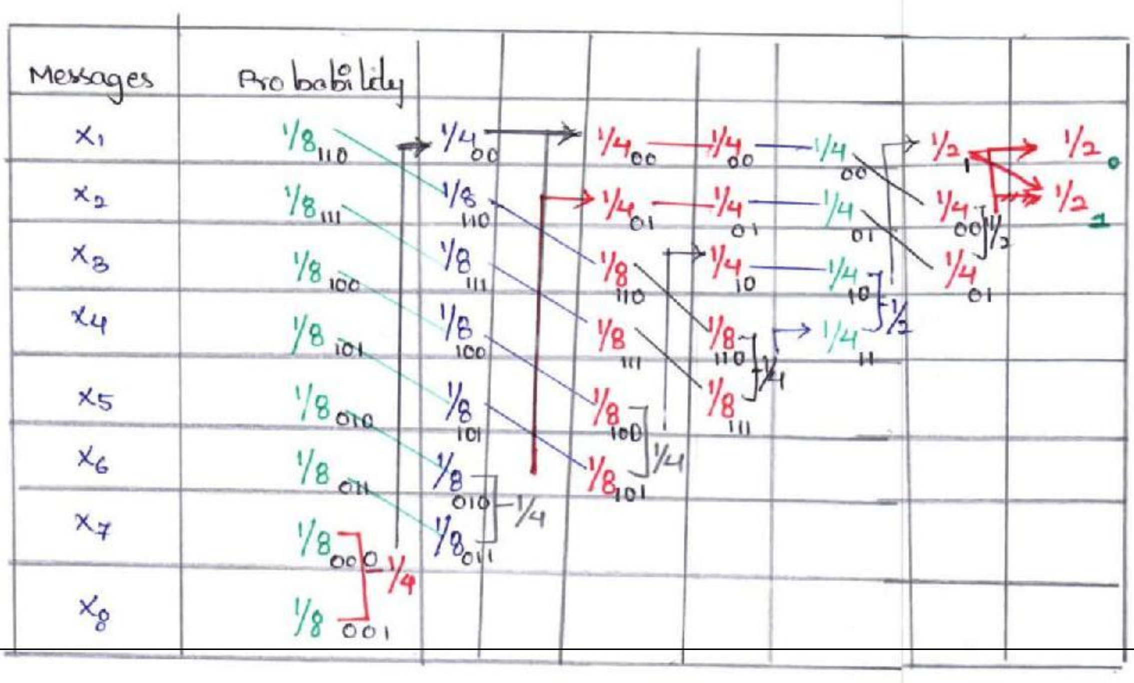
1) Shannon – fano Source encoding algorithm.
2) Hoffman source encoding algorithm and also calculate efficiency and redundancy in each of these case and compare them.

Given a DMS emits 8 equiprobable messages.

H = $\log_2 M$ = log 2^8 = 3 bits / message.

Symbols	probability	Source code word	Length
X_1	1/8		
X_2	1/8		
X_3	1/8		
X_4	1/8		
X_5	1/8		
X_6	1/8		
X_7	1/8		
X_8	1/8		

→ Construction of source code words using Hoffman source encoding technique:



Steps:

1) Arrange the given set of messages in the order of decreasing probability.
2) Add last two messages by means of adding their probability in this process a new message is obtained.
3) Now arrange messages in the order of decreasing probability.
4) Repeat step 2 & 3 till the last column contains only two messages.

Symbols	probability	Source code word	Length(l_k)
X_1	1/8	1 1 0	3
X_2	1/8	1 1 1	3
X_3	1/8	1 0 0	3
X_4	1/8	1 0 1	3
X_5	1/8	0 1 0	3
X_6	1/8	0 1 1	3
X_7	1/8	0 0 0	3
X_8	1/8	0 0 1	3

Efficiency:

$$\boxed{\eta = \frac{H}{L} \times 100}$$

$$\sum_{k=1}^{8} P_k l_k$$

$$= \frac{3}{3} \times 100$$

$$\eta = 100\%$$

REDUNDANCY:

$$\boxed{R = 1 - \frac{H}{L}}$$

(n,k) linear block codes: / cyclic code:

→ Matrix description of (n,k) linear block codes.

o/p $[c]_{1\times n} = [c]_{1\times k} \cdot [G]_{k\times n}$

G = Generator matrix
D = data word
(Message word)
(Source code word)

$[\, C_1, C_2, C_3 \ldots C_k, C_1, C_2 \ldots C_q] = [\, d_1, d_2, \ldots d_k]_{1\times K}$

$$\begin{pmatrix} \vdots \\ I_K : p_{log} \\ \vdots \\ \end{pmatrix}_{k \times q}$$

2

K –message bits q – check bits

$N = k + q$

I_k ; Identity matrix of order ‘K’

$P_{k \times q}$: Arbitrary matrix.

$$[P]_{k\times q} = \begin{pmatrix} P_{11} & P_{12} & p_{13} \dots & p_{1q} \\ P_{11} & P_{12} & p_{13} \dots & p_{1q} \\ : & & & \\ P_{11} & P_{12} & p_{13} \dots & p_{1q} \end{pmatrix}_{k \times q}$$

→ A generator matrix for generating linear codes is as follows.

$$G = \begin{array}{c} \quad I_4 \qquad\qquad\qquad\qquad p_{4\times 3} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & : & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & : & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & : & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & : & 1 & 1 & 1 \end{pmatrix} \end{array}$$

→ Find the code words for the possible messages

→ The above is generator matrix for a (7 , 4) linear block code.

Possible message words = $2^k = 2^4 = 16$

$[D_0]_{1\times K}$ = 0 0 0 0

$[D_1]_{1\times K}$ = 0 0 0 1

$[D_2]_{1\times K}$ = 0 0 1 0

$[D_3]_{1\times K}$ = 0 0 1 1

$[D_4]_{1\times K}$ = 0 1 0 0

$[D_5]_{1\times K}$ = 0 1 0 1

$[D_6]_{1\times K}$ = 0 1 1 0

$[D_7]_{1\times K}$ = 0 1 1 1

$[D_8]_{1\times K}$ = 1 0 0 0

$[D_9]_{1\times K}$ = 1 0 0 1

$[D_{10}]_{1\times K}$ = 1 0 1 0

$[D_{11}]_{1\times K}$ = 1 0 1 1

$[D_{12}]_{1\times K}$ = 1 1 0 0

$[D_{13}]_{1\times K}$ = 1 1 0 1

$[D_{14}]_{1\times K}$ = 1 1 1 0

$[D_{15}]_{1\times K}$ = 1 1 1 1

Possible code words

$[C]_{1\times 7} = [D_0]\,[G]$

$$= [\,0\ 0\ 0\ 0\,] \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$= [\,0\ 0\ 0\ 0\ 0\ 0\ 0\,]$

$[C]_{1\times 7} = [D_7]\,[G]$

$$= [\,0\ 1\ 1\ 1\,] \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

= [0 1 1 1 0 0 1] + → +

(n,k) linear block code is known as hamming code if it satisfies

ERROR DETECTION AND ERROR CORRECTION CAPABILITIES OF LINEAR BLOCK CODES:

→ Let 'C' be the transmitted code word.

→Let 'R' is received code word.

Case(i):

R = C

→ Received code word has no error.

→For error detection and error correction calculate syndrome vector.

$S_{1\times q} = R_{1\times n} H^T_{n\times q}$

H^T : Transpose of parity check matrix.

Construction of

parity check matrix [H] = [pT

pT : Transpose of orbitary matrix.

Construction of generator matrix

→ If syndrome vector is zero that means there is no error in the received code word,

Case (ii):

R ≠ C

⇒ R = C + E

E → error

C → Transmitted code word.

→ $S = RH^T$

$= [C \oplus E] H^T$

$= CH^T \oplus EH^T$

→Syndrome vector is a non zero vector, which means there is a error in the received code word.

$= 0 \oplus EH^T$ $(\because CH^T = 0)$

→ For a (7 ,4) linear block code 'G' is given by

$$G = \left(\begin{array}{cccc:ccc} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ & I_4 & & & & p_{4\times q} & \\ & & & & & 4\times 3 & \end{array}\right)_{4\times 7}$$

Find (1) All possible code words for all possible message words.

2) Find syndrome vector for received code word

R = 0111001

3) Find syndrome vector for a received code word Type equation here.

R = 0 0 1 1 0 0 1

$$[p]_{4\times 3} = \begin{matrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}_{k\times q}$$

4×3

$$P^T = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix}_{3\times 4}$$

$H = [P^T \vdots I_q]$

$$= \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} 3\times 7$$

<u>**(b) R = 0 1 1 1 0 0 1**</u>

$S = RH^T$

$$= [\,0\ 1\ 1\ 1\ 0\ 0\ 1\,] \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$[0\oplus 1\oplus 0\oplus 1\oplus 0\oplus 0\oplus 0 \quad 0\oplus 0\oplus 1\oplus 1\oplus 0\oplus 0\oplus 0 \quad 0]$

No Error

$\therefore S = 0$

$\Rightarrow R = C$

$= [0 \quad 0 \quad 0] = [0]_{1\times 3}$

c) **R = [0 0 1 1 0 0 1]**

$S = RH^T$

$$= [\,0\ 1\ 1\ 1\ 0\ 0\ 1\,] \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$S = [1\ 0\ 1]$ error is present in bit no 2

$\therefore S = 101$

$\Rightarrow R \neq C$

$\therefore$ Corrected code word

$[0 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 1]$ $\rightarrow\therefore$ Syndrome vector matches with 2^{nd} Row of H^T

$\therefore$ Error bit is 2^{nd} bit

$\rightarrow$ An error control code has the following parity check matrix.

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ & & & & & \\ & & & & & \end{pmatrix}$$

$$H = \begin{matrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{matrix} \quad \begin{matrix} 3\times 6 \\ q\times n \end{matrix}$$

1) Determine generator matrix "G"

2) Decode the received code word 110110, Comment on error detection capabilities

$$H = [\, P^T \vdots I_q \,]_{q\times n}$$

$$G = [\, I \vdots P \,]_{k\times n}$$

$$\therefore \quad P^T = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$\Rightarrow P = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}_{3\times 3} \; k\times q$$

→ Generator matrix

$$G = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} \quad \begin{matrix} 3\times 6 \\ K\times n \end{matrix}$$

b) Received Code word

$R = 1\ 1\ 0\ 1\ 1\ 0$

$S = RH^T$

$$S = [\, 1\ 1\ 0\ 1\ 1\ 0 \,] \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$= [\, 0\ 1\ 1 \,]$

∴ The syndrome matrix matches with 2nd Row of H^T matrix.

∴ The 2nd bit in the received code word has error

⇒ Corrected Code Word

$R = 1\ 0\ 0\ 1\ 1\ 0$

What is a systematic block code? What is a syndrome vector low is it useful?

A linear (n,k) block code has a generator matrix.

$$G = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

1) Find all its code words.

2) Find its H matrix

3) What is the minimum distance of code and its error correcting capacity.

$$\text{Given } G = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} k\times n$$

$K = 2$

$n = 4$

$q = 2.$

→ Possible message code words $= 2^k$

4

6

$= 2^2$

$\mathbf{= 4}$

→They are 00,01,10,11

a) Code Words:

$[C] = [D]\ [G]$

$[C_0] = [\ 0\ 0\ ] \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}$

$= [\ 0\ 0\ 0\ 0]$ $\quad \omega(C_0) = 0$

$[C_1] = [\ 0\ 1\ ] \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}$

$= [\ 0\ 1\ 0\ 1]$ $\quad \omega(C_1) = 2$

$[C_2] = [\ 1\ 0\ ] \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}$ distance b/w C_0 & C_1

$d(C_0, C_1) = 0 + 1 + 0 + 1 = 2$

$= [\ 1\ \ 0\ \ 1\ \ 1\ ]$ $\quad \omega(C_3) = 2$

$[C_3] = [\ 1\ 1\ ] \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}$

$= [\ 1\ \ 1\ \ 1\ \ 0]$ $\quad \omega(C_4) = 3$

Minimum weight → 2

b) H – Matrix:

$H = [\ P^T \vdots I_q]_{q\times n}$

$G = [\ I \vdots P\ ]$

$\therefore \mathbf{P} = \begin{pmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

$\Rightarrow \mathbf{P^T} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} \end{pmatrix}$

$\therefore \mathbf{H} = \begin{pmatrix} \mathbf{1} & \mathbf{0} & \vdots & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \vdots & \mathbf{0} & \mathbf{1} \end{pmatrix}_{2\times 4}$

→ The P format of a systematic block code is

$C = [K \vdots q\ ]$

Where K = message bits

q = check bits

Linearity:

→ Block code satisfies linearity property.

→ Linearity property states that "sum of two code vectors is a valid code vector"

Ex: $C_1 (+) C_2 = [\ 0\ 1\ 0\ 1\ ] (+) [\ 1\ 0\ 1\ 1]$

$= [\ 1\ 1\ 1\ 0\ ]$

$= C_3$

→ "Weight" of code word is no. of non – zero elements present in code word.

→ If HBC satisfies then it is known as binary code.

→The parity check bits (8,4) block code are generated by

$C_5 = d_1 + d_2 + d_4$

$C_6 = d_1 + d_2 + d_3$

$C_5 = d_1 + d_3 + d_4$

$C_5 = d_2 + d_3 + d_4$

Where d_1, d_2, d_3 and d_4 are message bits.

Find a) Generator matrix

b) Parity check matrix

c) Minimum weight of this code.

Given (8 , 4) linear block codes

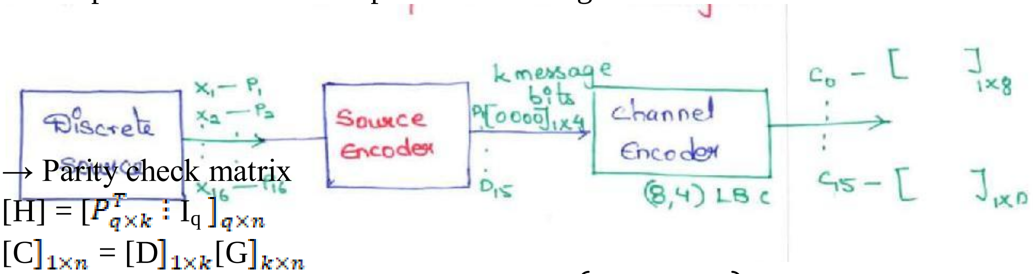
⇒ n = 8 bits

K = 4 bits ⟶ message bits

No.of message words = $2^k = 2^4 = 16$

⟶ Distance b/w code words:

No. of positions in which respective bits disagree.



⟶ Parity check matrix

$[H] = [P^T_{q\times k} \vdots I_q]_{q\times n}$

$[C]_{1\times n} = [D]_{1\times k}[G]_{k\times n}$

$[\underbrace{C_1\, C_2\, C_3\, C_4}_{\text{Message bits}}\;\; \underbrace{C_5\, C_6\, C_7\, C_8}_{\text{Check bits}}] = [d_1\, d_2\, d_3\, d_4] \left(I_4 \vdots P_{4\times4} \right)$

$$= [d_1\, d_2\, d_3\, d_4] \begin{pmatrix} 1 & 0 & 0 & 0 & P_{11} & P_{12} & P_{13} & P_{14} \\ 0 & 1 & 0 & 0 & P_{21} & P_{22} & P_{23} & P_{24} \\ 0 & 0 & 1 & 0 & P_{31} & P_{32} & P_{33} & P_{34} \\ 0 & 0 & 0 & 0 & P_{41} & P_{42} & P_{43} & P_{44} \end{pmatrix}$$

⇒ $C_1 = d_1$

$C_2 = d_2$

$C_3 = d_3$

$C_4 = d_4$

$C_5 = d_1 + d_2 + d_4$

$C_6 = d_1 + d_2 + d_3$

$C_7 = d_1 + d_3 + d_4$

$C_8 = d_2 + d_3 + d_4$

$$\Rightarrow P = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix}$$

Generator matrix G = [I ⋮ P]

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}$$

Parity check matrix H = [p^T I]

$$H = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

HOMMING CODES:

⟶ Hamming codes are subclass of (n,k) linear block codes.

⟶ Those codes satisfy following conditions.

1) No. of check bits

$q \geq 3$

2) Block length

$\boxed{n = 2^q - 1}$

3) No. of message bits

$\boxed{K = n - q}$

4) Minimum distance

$\boxed{D_{min} = 3}$

→ We know that code rate (for a channel encoding technique) such as (n,k) linear block code / convolution code is given by

$$r = \frac{k}{n}$$

$$= \frac{n-q}{n}$$

$$= 1 - \frac{q}{n} \qquad = 1 - \frac{q}{2^q - 1}$$

→ From the above equation we observe that

$r = 1$; if $q >> 1$

ERROR DETECTION & ERROR CORRECTION CAPABILITIES OF HAMMING CODE:

→ The minimum weight of the code is "3"

3 = Min hamming weight

Since

$d_{min} \rightarrow$ Minimum hamming distance

$\boxed{d_{min}}$

$\boxed{d_{min} = [w(x)]_{Min}}$

$[w(x)]_{Min} \rightarrow$ minimum Hamming weight

$d_{min} \rightarrow$ minimum Hamming distance

Where x → Code vector

$x \neq [0\ 0 \ldots 0]$

→ The error detection and correction capabilities of a hamming code.

$\boxed{d_{min} \geq S+1}$

Where S → No. of errors can be detected

$3 \geq S + 1$

$\Rightarrow S \leq 2$

Thus two errors can be detected

And

$\boxed{d_{min} \geq 2t + 1}$

Where t → No. of errors that can be corrected

$3 \geq 2t + 1$

$t \leq 1$

BINARY CYCLIC CODES:

→ Cyclic codes are subclass of linear block codes.

→ Cyclic codes can be systematic / non systematic.

$\boxed{X = (M : C)}$

X → Code word

C → Chek bits

M → Message bits

DEFINATION OF CYCLIC CODE:

→ A linear block code is said to cyclic code if every cyclic shift of code vector produces a valid

code vector.
→ The properties of cyclic codes.
1) Linearity property
2) Cyclic property

Cyclic property:

Very cyclic shift of a code vector produces another valid code vector. Because of this property name cyclic is given.
→ Consider n bit code Vector as shown below
$X = \{ x_{n-1}, x_{n-2} \ldots . x_1 , x_0\} = x_{n-1}\, x^{n-1} + x_{n-2}\, x^{n-2} + \ldots\ldots + x_1x^1 + x_0x^0$
X → Code Vector
Where $x_{n-1} , x_{n-2} \ldots x_0$ → Elements of a code word/ vector.
→ Let us shift the code vector "x" cyclically to left side one cycle shift of x produces a another valid code vector
$X^1 = \{ x_{n-1}, x_{n-2} \ldots . x_1 , x_0\ x_{n-1}\}$

ALGEBRIC STRUCTURE OF BINARY CYCLIC CODES:

→ The code words can be represented by a polynomial.
Ex: Consider 'n' bit code vector.
$X = \{ x_{n-1}, x_{n-2} \ldots . x_1 , x_0 \}$
Where $x_{n-1}, x_{n-2} \ldots . x_0$ → Elements of a code word.
→ The above code word 'x' can be represented by a polynomial of degree $\propto$ n-1.

$$\boxed{X(p) = x_{n-1}\, p^{n-1} + x_{n-2}\, p^{n-2} + \ldots\ \ldots + x_1\, p^1 + x_0\, p^0}$$

X(P) → Code word polynomial
(or)
Code vector polynomial
(or)
Code polynomial

Where p → Arbitrary variable.

$$\boxed{X(x) = x_{n-1}\, p^{n-1} + x_{n-2}\, p^{n-2} + \ldots\ \ldots + x_1\, x^1 + x_0\, x^0}$$

Where X → Arbitrary variable
→ Here x(p) is a code polynomial of degree n-1.
Where 'p' is arbitrary variable of polynomial.
→ The power of 'p' represents the position of code word bits.
→ p^{n-1} → represent MSB
P^0 → LSB
P^1 → 2^{nd} bit from LSB side.
→ Cyclic code can be systematic (or) non systematic.

GENERATION OF CODE VECTORS IN NON- SYSTEMATIC FORM:

→ Let $M = \{ m_{k-1}, m_{k-2,} m_{k-3} \ldots . m_1 , m_0\}$
→ let x(p) represents code polynomial

$$\boxed{X(p) = M(p)\, G(p)}$$

G(p)→ Generator polynomial.
X(p) → Code polynomial
M(p) → Message polynomial

→ For an (n,k) cyclic code
q = n – k, represents of no. of check bits.
The generating polynomial is given as

$$\boxed{G(p) = p^q + g_{q-1}p^{q-1} + \ldots\ldots + g_1\, p^1 + 1.\, P^0}$$

Here $g_{q-1} , g_{q-2} \ldots\ldots g_1$ are coefficients of generator polynomial.
→ All the code vectors obtained for a (n, k) cyclic code satisfy cyclic property.

Note:

→ Generator polynomial remains same for all the code vectors.

→ The generator polynomial of a (7,4) cyclic code $G(p) = p^3 + p + 1$; find all the code vectors , for the code in non systematic form.

→ Give (7,4) cyclic code

⇒ n = 7 → No. of code bits

K = 4 → No. of message bits

Q = n-k = 3 → No. of check bits

→ No. of source code words possible

$= 2^4 = 16$

i.e., [0 0 0 0] to [1 1 1 1]

Message polynomial =

$M(p) = m_3p^3 + m_2p^2 + m_1p^1 + m_0p^0$

→ GENERATOR POLYNOMIAL

$G(p) = p^3 + p + 1$

Code polynomial

$X(p) = M(p). G(p)$

$= (m_3p^3 + m_3\ p2 + m_1p + m_0). (p^3 + p + 1)$

For M_q : → 1 0 0 1

$M(p) = p^3+1$

$x\ (p) = (p^3 + 1). (p^3+p+1)$

$= p^6 + p^4 + p^3 + p^3 + p + 1$

$= p^6 + p^4 + p + 1$

$X_q\ (p) = p^6 + p^4 + p + 1$

X = [1 0 1 0 0 1 1]

CONSTRUCTION OF SYSTEMATIC CODE VECTORS:

→ There are three steps in encoding process. For a systematic (n,k) cyclic code.

→ They are

1) Multiply message polynomial by p^{n-k} i.e., p^q

To obtain p^q M(p).

2) Divide p^q M(p) by G(p) to obtain remainder polynomial

C(p) → Remainder polynomial / check bits polynomial.

3) To obtain code polynomial x(p). Add check bits polynomial C(p) to p^qM(p)

{{{{{ $X(p) = p^q M(p)\ c(p)$

c(p) → Check bits polynomial.

M(p) → Message polynomial.

X(p) → Code polynomial in systematic form.

P → The generator polynomial for a (7,4) cyclic code G (p) = $1 + p + p^3$. Determine all the systematic code vectors.

→ Given (7,4) cyclic code

n = 7 → No. of code bits

k = 4 → No. of message bits

q = 3 → No. of check bits.

→ No. of message words = $2^k = 2^4 = 16$

M_0 = [0 0 0 0]

⋮

M_{15} = [1 1 1 1]

→ Message polynomial

$M(p) = m_3p^3 + m_2p^2 + m_1p^1 + m_0p^0$

→Generator polynomial

$G(p) = p^3+p+1$

Let for M_q = [1 0 0 1]

$M_q = p^3+1$

1) $p^qM(p) = p^3\ M(p)$

$= p^3\ (p^3+1) = P^6+p^3$

2)

$$\boxed{\frac{p^qM(p)}{G(p)}} = \frac{p^6+p^3}{p^3+p+1}$$

$P^3+P+1)\ \ p^6 + p^3\ \ (p^3+p$

$p^6 + p^4 + p^3$

$+ \quad + \quad +$

p^4

P^4+P^2+P

$P^2 + P$

⇒ $C(p) = P^2+P$

3) $x(p) = p^q\ M(p) \oplus p^2+p$

X_q = [1 0 0 1 1 1 0]

Message bits Check bits

Systematic. Code vector.

Convolution codes:

→ in block codes, the encoder accepts 'k' bits message block and generates, associated code word of length 'n' bits.

→ in block codes code words are produced on block by block basis.

Definition:

→ in convolution coding code words are generated by convolving input message bits with generator. Hence convolution codes.

→ Convolutional encoder operates on continuous i/p bit stream (message bit sequence)

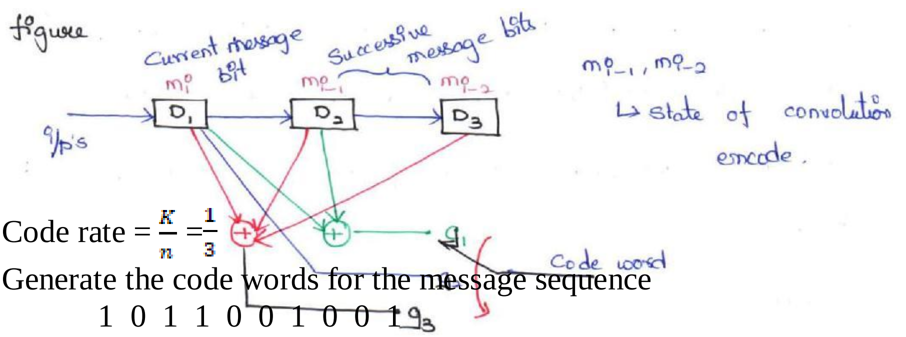
→ Realization of convoltional encoder:

→Convolutional encoder realized with fixed length shift register (to store i/p bit stream) & with the help of module to adders (equivalent to binary convolution).

→code rate of a convolutional encoder.

$r = \frac{K}{n}$

→ **sketch the code free for the convolutional encoder shown in figure**



Code rate = $\frac{K}{n} = \frac{1}{3}$

Generate the code words for the message sequence

1 0 1 1 0 0 1 0 0 1

Generator vectors

$g_1 = D_1\ \ D_2$

$g_2 = D_1$

$g_1 = D_1 (+) D_2 (+) D_3$

$g^{(1)} = (1, 1, 0)$

$g^2 = (1, 0, 0)$

$g^3 = (1, 1, 1)$

→ MSB enters first

Initially $D_1 = D_2 = D_3 = 0$

⟹ $g_1 = g_2 = g_3 = 0$

generator polynomials
(impulse response)

$g^1(x) = 1.x^0 + 1.x^1 + D.x^2$
$= 1 + x$
$g^2(x) = 1$
$g^3(x) = 1 + x + x^2$

1) $D_1\ D_2\ D_3 = 1\ 0\ 0$
$g_1\ g_2\ g_3 = 1\ 1\ 1$
2) $D_1\ D_2\ D_3 = 0\ 1\ 0$
$g_1\ g_2\ g_3 = 1\ 0\ 1$
3) 0 0 1 ⟹ 0 0 1

	D_1	D_2	D_3	$g^1 = D_1 + D_2$	$g^2 = D_1$	$g^3 = D_1 + D_2 + D_3$	Code word $g^1\ g^2\ g^3$
initially	0	0	0	0	0	0	0 0 0
1 →	1	0	0	1	1	1	1 1 1
0 →	0	1	0	1	0	1	1 0 1
1 →	1	0	1	1	1	0	1 1 0
1 →	1	1	0	0	1	0	0 1 0
0 →	0	1	1	1	0	0	1 0 0
0 →	0	0	1	0	0	1	0 0 1
1 →	1	0	0	1	1	1	1 1 1
0 →	0	1	0	1	0	1	1 0 1
0 →	0	0	1	0	0	1	0 0 1
1 →	1	0	0	1	1	1	1 1 1

→ The functionality of a convolution encoder can be graphically represented as
1) Code Tree
2) State diagram
3) Trellis diagram

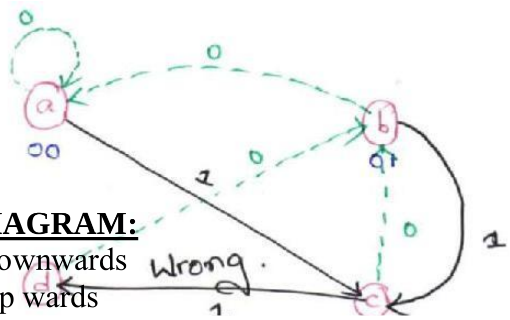
STATE DIAGRAM:

→ m_{i-1}, m_{i-2} are states of convolutional codes.

⇒ foure states are. Possible

a – 0 0 1 ________
b – 0 1 0 - - - - - - - -
c – 1 0
d – 1 1

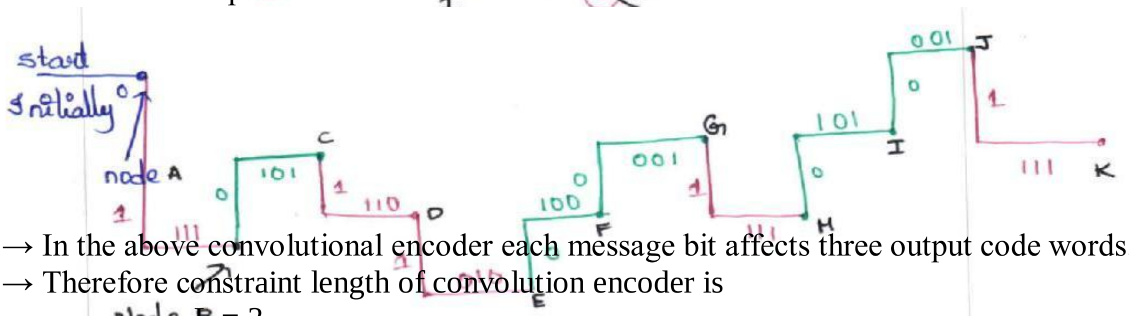
Current state		next state	
0	0	0	0
0	1	0	1
1	0	1	0
1	1	1	1



CODE TREE DIAGRAM:

For i/p bit 1 → downwards

bit 0 → up wards



→ In the above convolutional encoder each message bit affects three output code words.

→ Therefore constraint length of convolution encoder is

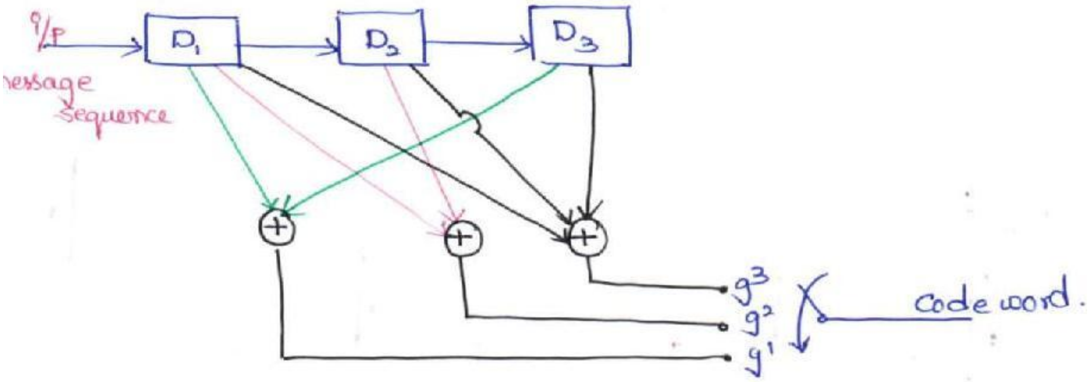
= 3

→ Convolutional encoder has a signal shift register with two stages i.e. constraint length k = 3, 3 modulo two address and an o/p mwc, the generator sequence of encoder are as follows.

$g^1 = (1, 0, 1)$

$g^2 = (1, 1, 0)$

$g^3 = (1, 1, 1)$ Draw the block diagram of encoder.



Linear Block Codes

In coding theory, a block code is any member of the large and important family of error-correcting codes that encode data in blocks. There is a vast number of examples for block codes, many of which have a wide range of practical applications. Block codes are conceptually useful because they allow coding theorists, mathematicians, and computer scientists to study the limitations of *all* block codes in a unified way. Such limitations often take the form of *bounds* that relate different parameters of the block code to each other, such as its rate and its ability to detect and correct errors. Examples of block codes are Reed–Solomon codes, Hamming codes, Hadamard codes, Expander codes, Golay codes, and Reed–Muller codes. These examples also belong to the class of linear codes, and hence they are called linear block codes.

- We will consider only binary data
- Data is grouped into blocks of length *k* bits (dataword)
- Each dataword is coded into blocks of length *n* bits (codeword), where in general *n*>*k*
- This is known as an (*n*,*k*) block code
- A vector notation is used for the datawords and codewords,
  - Dataword d = (*d1 d2….dk*)
  - Codeword c = (*c1 c2……..cn*)
- The redundancy introduced by the code is quantified by the code rate,
  - Code rate = *k*/*n*
  - i.e., the higher the redundancy, the lower the code rate

Hamming Distance:

- Error control capability is determined by the Hamming distance
- The Hamming distance between two codewords is equal to the number of differences between them, e.g.,

  10011011

  11010010 have a Hamming distance = 3
- The maximum number of detectable errors is:

$$d_{\min} - 1$$

- That is the maximum number of correctable errors is given by ,r= $[d_{min}-1]/2$

where *d*min is the minimum Hamming distance between 2 codewords and means the smallest integer.

Linear Block Codes:

As seen from the second Parity Code example, it is possible to use a table to hold all the codewords for a code and to look-up the appropriate codeword based on the supplied dataword Alternatively, it is possible to create codewords by addition of other codewords. This has the advantage that there is now no longer the need to held every possible codeword in the table. If there are k data bits, all that is required is to hold k linearly independent codewords, i.e., a set of k codewords none of which can be produced by linear combinations of 2 or more codewords in the set.The easiest way to find k linearly independent codewords is to choose those which have '1' in just one of the first k positions and '0' in the other k-1 of the first k positions.

For example for a (7,4) code, only four codewords are required, e.g.,

$$\begin{array}{ccccccc} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{array}$$

So, to obtain the codeword for dataword 1011, the first, third and fourth codewords in the list are added together, giving 1011010. This process will now be described in more detail

An (n,k) block code has code vectors d=($d1d2….dk$) and c=($c1$ $c2$……..cn

The block coding process can be written as c= dG where G is the Generator Matrix

$$\mathbf{G}=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ . & . & \cdots & . \\ a_{k1} & a_{k2} & \cdots & a_{kn} \end{bmatrix}=\begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ . \\ \mathbf{a}_k \end{bmatrix}$$

- Thus,

$$\mathbf{c}=\sum_{i=1}^{k} d_i \mathbf{a}_i$$

ai must be linearly independent, i.e.,

Since codewords are given by summations of the ai vectors, then to avoid 2 datawords having the same codeword the ai vectors must be linearly independent.

• Sum (mod 2) of any 2 codewords is also a codeword, i.e.,Since for datawords d1 and d2 we have

$d_3=d_1+d_2$

So,

$$c_3=\sum_{i=1}^{k} d_{3i}a_i=\sum_{i=1}^{k}(d_{1i}+d_{2i})a_i=\sum_{i=1}^{k} d_{1i}a_i+\sum_{i=1}^{k} d_{2i}a_i$$

$C_3=C_1+C_2$

Error Correcting Power of LBC:

• The Hamming distance of a linear block code (LBC) is simply the minimum Hamming weight (number of 1's or equivalently the distance from the all 0 codeword) of the non-zero codewords

• Note d(c1,c2) = w(c1+ c2) as shown previously

• For an LBC, c1+ c2=c3

• So min (d(c1,c2)) = min (w(c1+ c2)) = min (w(c3))

• Therefore to find min Hamming distance just need to search among the 2*k* codewords to find the min Hamming weight – far simpler than doing a pair wise check for all possible codewords.

Creating block codes: The block codes are specified by (n.k). The code takes k information bits and computes (n-k) parity bits from the code generator matrix. Most block codes are systematic in that the information bits remain unchanged with parity bits attached either to the front or to the back of the information sequence. *Hamming code, a simple linear block code *Hamming codes are most widely used linear block codes. * A Hamming code is generally specified as (2n- 1, 2n-n-1). * The size of the block is equal to 2n-1. * The number of information bits in the block is equal to 2n-n-1 and the number of overhead bits is equal to n. All Hamming codes are able to detect three errors and correct one.

Reed-Solomon Codes: Reed Solomon (R-S) codes form an important sub-class of the family of Bose- Chaudhuri-Hocquenghem (BCH) codes and are very powerful linear non-binary block codes capable of correcting multiple random as well as burst errors. They have an important feature that the generator polynomial and the code symbols are derived from the same finite field. This enables to reduce the complexity and also the number of computations involved in their implementation. A large number of R-S codes are available with different code rates. An R-S code is described by a generator polynomial g(x) and other usual important code parameters such as the number of message symbols per block (k), number of code symbols per block (n), maximum number of erroneous symbols (t) that can surely be corrected per block of received symbols and the designed minimum symbol Hamming distance (d). A parity-check polynomial h (X) of order k also plays a role in designing the code. The symbol x, used in polynomials is an indeterminate which usually implies unit amount of delay.

For positive integers m and t, a primitive (n, k, t) R-S code is defined as below: Number of encoded symbols per block: n = 2m – 1 Number of message symbols per block: k Code rate: R= k/n Number of parity symbols per block: n

– k = 2t Minimum symbol Hamming distance per block: d = 2t +1. It can be noted that the block length n of an (n, k, t) R-S code is bounded by the corresponding finite field GF(2m). Moreover, as n – k = 2t, an (n, k, t) R-S code has optimum error correcting capability.

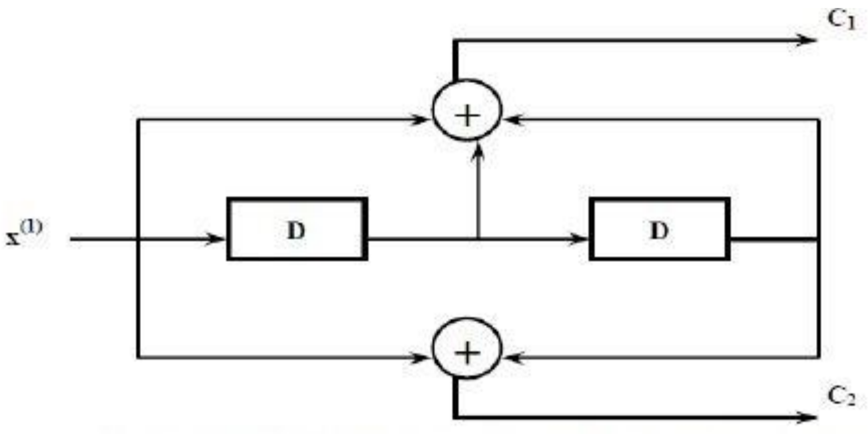
Convolutional codes:

Convolutional codes are widely used as channel codes in practical communication systems for error correction. The encoded bits depend on the current k input bits and a few past input bits. The main decoding strategy for convolutional codes is based on the widely used Viterbi algorithm.

Convolutional codes are commonly described using two parameters: the code rate and the constraint length. The code rate, k/n, is expressed as a ratio of the number of bits into the convolutional encoder (k) to the number of channel symbols output by the convolutional encoder (n) in a given encoder cycle.

The constraint length parameter, K, denotes the "length" of the convolutional encoder, i.e. how many k-bit stages are available to feed the combinatorial logic that produces the output symbols. Closely related to K is the parameter m, which can be thought of as the memory length of the encoder.

A simple convolutional encoder is shown below fig . The information bits are fed in small groups of k-bits at a time to a shift register. The output encoded bits are obtained by modulo-2 addition (EXCLUSIVE-OR operation) of the input information bits and the contents of the shift registers which are a few previous information bits



A convolutional encoder with k=1, n=2 and r=1/2

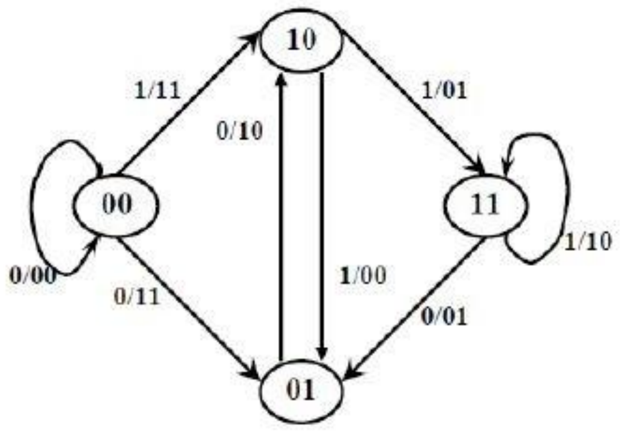
The operation of a convolutional encoder can be explained in several but equivalent ways such as, by

a) state diagram representation.

b) tree diagramrepresentation.

c) trellis diagram representation

a) **State Diagram Representation:** A convolutional encoder may be defined as a finite state machine. Contents of the rightmost (K-1) shift register stages define the states of the encoder. So, the encoder in the above **Fig** has four states. The transition of an encoder from one state to another, as caused by input bits, is depicted in the state diagram.**Fig. 3.2** shows the state diagram of the encoder in **Fig** above A new input bit causes a transition from one state to another

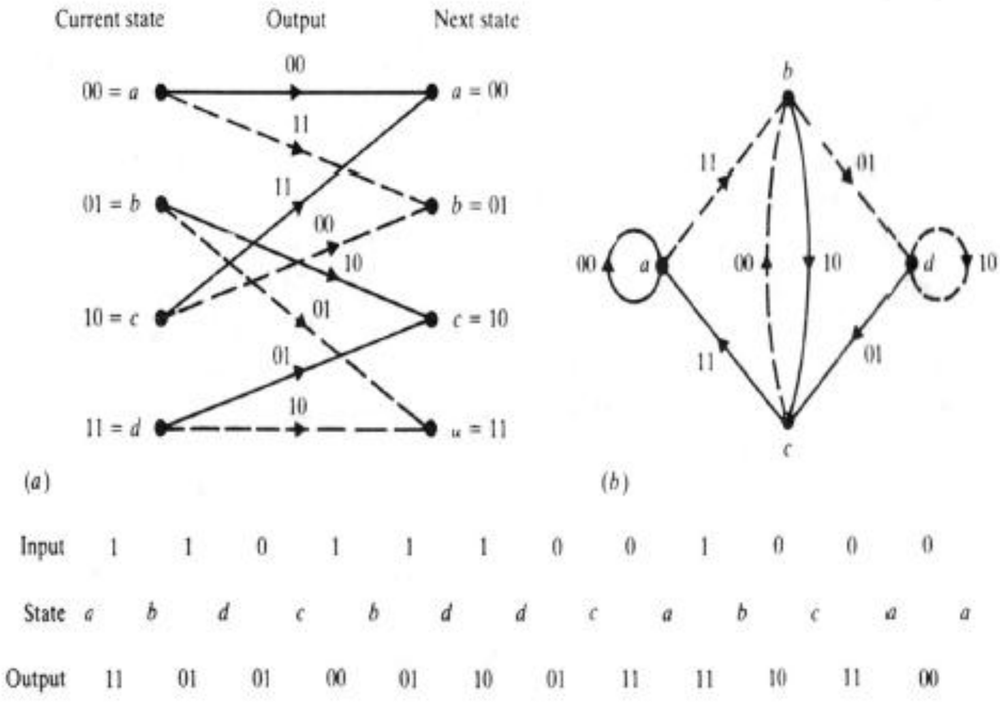


State diagram representation for the encoder

b) **Tree Diagram Representation:** The tree diagram representation shows all possible information and encoded sequences for the convolutional encoder. **Fig. 3.3** shows the tree diagram for the encoder in **Fig. 3.1**. The encoded bits are labeled on the branches of the tree. Given an nput sequence, the encoded sequence can be directly read from the tree.

Representing convolutional codes compactly: code trellis and state diagram:

State diagram

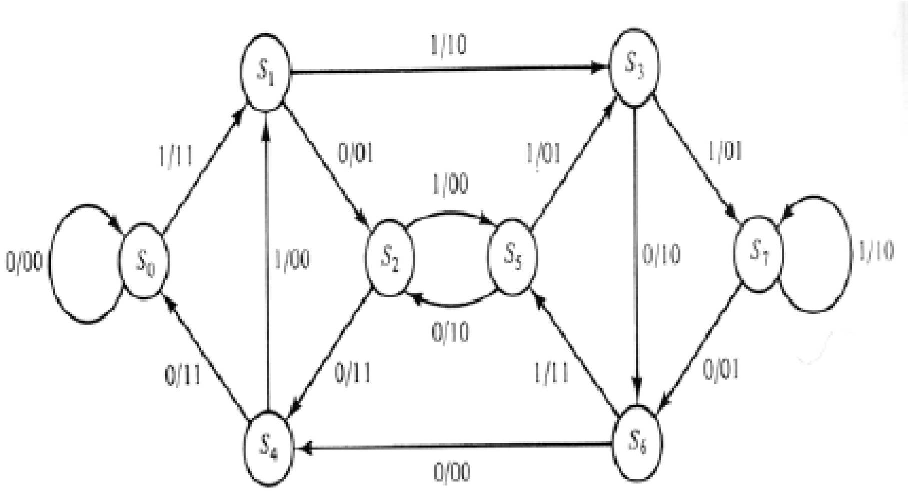


Inspecting state diagram: Structural properties of convolutional codes:

• Each new block of k input bits causes a transition into new state

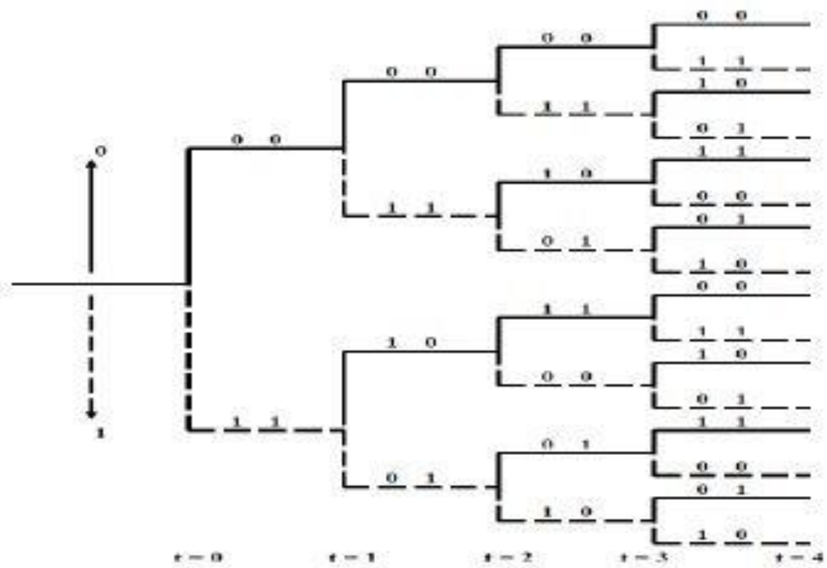
• Hence there are $2k$ branches leaving each state

• Assuming encoder zero initial state, encoded word for any input of k bits can thus be obtained. For instance, below for **u**=(1 1 1 0 1), encoded word **v**=(1 1, 1 0, 0 1, 0 1, 1 1, 1 0, 1 1, 1 1) is produced:



Encoder state diagram for $(n,k,L)=(2,1,2)$ code - note that the number of states is $2L+1 = 8$

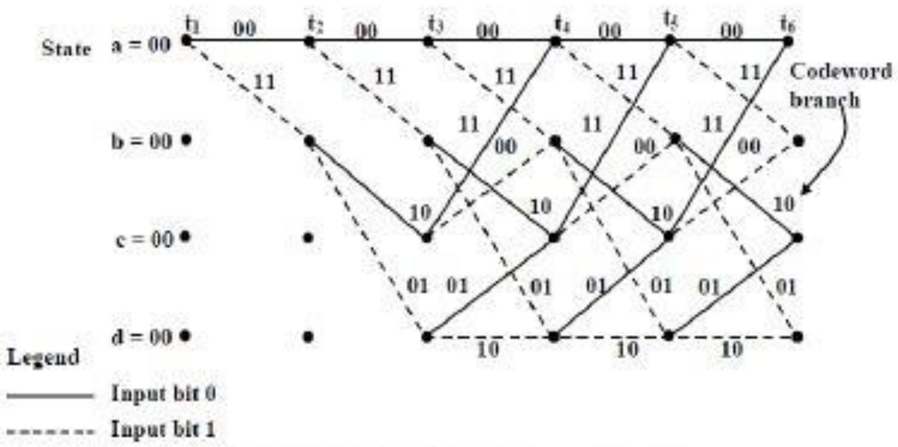
n	k	R_c	L	d_f	$R_c\, d_f/2$
4	1	1/4	3	13	1.63
3	1	1/3	3	10	1.68
2	1	1/2	3	6	1.50
			6	10	2.50
			9	12	3.00
3	2	2/3	3	7	2.33
4	3	3/4	3	8	3.00



A tree diagram for the encoder

The above figure shows the tree diagram for the above encoder.

***c)* Trellis Diagram Representation:** The trellis diagram of a convolutional code is obtained from its state diagram. All state transitions at each time step are explicitly shown in the diagram to retain the time dimension, as is present in the corresponding tree diagram. Usually, supporting descriptions on state transitions, corresponding input and output bits etc. are labeled in the trellis diagram. It is interesting to note that the trellis diagram, which describes the operation of the encoder, is very convenient for describing the behavior of the corresponding decoder, especially when the famous „Viterbi Algorithm (VA)‟ is followed. The Fig.below shows the trellis diagram for the above encoder .



Trellis diagram for the encoder

Cyclic Codes

In coding theory, a **cyclic code** is a block code, where the circular shifts of each codeword gives another word that belongs to the code. They are error-correcting codes that have algebraic properties that are convenient for efficient error detection and correction.

Description of Cyclic Codes

• If the components of an n-tuple $\boldsymbol{v} = (v0, v1, \dots, vn-1)$ arecyclically shifted i places to the right, the resultant n-tuple would be

$\boldsymbol{v}^{(i)} = (v_{n-i}, v_{n-i+1}, \dots\dots\dots, v_{n-1}, v_0, v_1, \dots, v_{n-i-1})$.

Cyclically shifting $\boldsymbol{v}$ i places to the right is equivalent to cyclically shifting $\boldsymbol{v}$ $n-i$ places to the left.An (n, k) linear code $\boldsymbol{C}$ is called a *cyclic code* if every cyclic shift of a code vector in $\boldsymbol{C}$ is also a code vector in $\boldsymbol{C}$.

Code polynomial $\boldsymbol{v}(x)$ of the code vector $\boldsymbol{v}$ is defined as

$\boldsymbol{v}(x) = v0 + v1x + \cdot\cdot\cdot + vn-1\, x^{n-1}$.

• $\boldsymbol{v}^{(i)}(x) = x^i\, \boldsymbol{v}(x) \bmod x^n + 1$.

Proof: Multiplying $\boldsymbol{v}(x)$ by xi, we obtain

$x^i\, \boldsymbol{v}(x) = v_0x^i + v_1xi^{+1} + \cdot\cdot\cdot + {}_{vn-i-1}{}^{xn-1} + \cdot\cdot\cdot + v_{n-1}\, x^{n+i-1}$.

Then we manipulate the equation into the following form:

$x^i\boldsymbol{v}(x) = {}_{vn-i} + v_{n-i+1x} + \cdot\cdot\cdot + v_{n-1}\, x\, i^{-1} + v_0x^i + \cdot\cdot\cdot$

$+v_{n-i-1}\, x_{n-1} + v_{n-i}(x^n + 1) + v_{n-i-1}\, x(x^n + 1)$

$+ \cdot\cdot\cdot + v_{n-1}{}^{xi-1}(x^n + 1)$

$= \boldsymbol{q}(x)(x^n + 1) + \boldsymbol{v}^{(i)}(x)$,

where $\boldsymbol{q}(x) = v_{n-i} + v_{n-i+1}\, x + \cdot\cdot\cdot + v_{n-1}\, x^{i-1.}$

• The nonzero code polynomial of minimum degree in a cyclic code $\boldsymbol{C}$ is unique.

• Let $\boldsymbol{g}(x) = g_0 + g_1x + \cdot\cdot\cdot + g_{r-1}\, x^{r-1} + x^r$ be the nonzero code polynomial of minimum degree in an (n, k) cyclic code $\boldsymbol{C}$. Then the constant term $g0$ must be equal to 1.

Proof: Suppose that $g0 = 0$. Then

$\boldsymbol{g}(x) = g_1x + g_2x^2 + \cdot\cdot\cdot + g_{r-1}\, x^{r-1} + x^r$

$= x(g_1 + g_2x + \cdot\cdot\cdot + g_{r-1}\, x^{r-2} + x^{r-1})$.

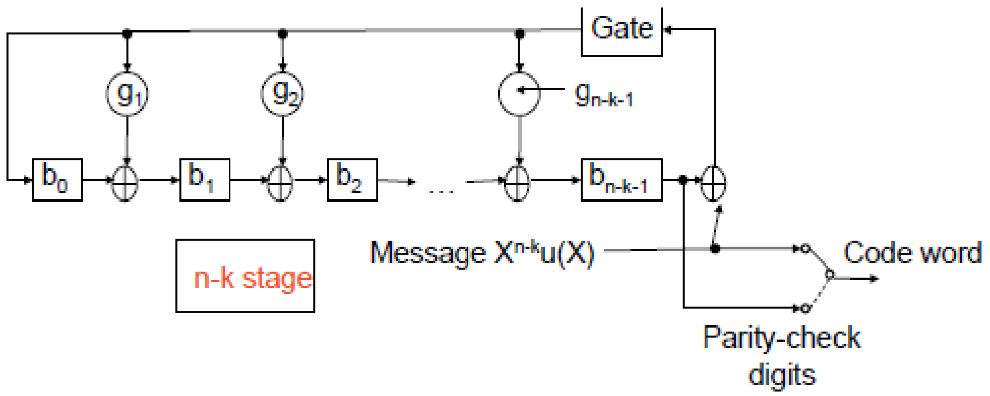
If we shift $\boldsymbol{g}(x)$ cyclically $n-1$ places to the right (or one place to the left), we obtain a nonzero code polynomial, $g_1 + g_2x + \cdot\cdot\cdot + g_{r-1}\, x^{r-2} + x^{r-1}$, which has a degree less than r.

A (7, 4) Cyclic Code Generated by $\boldsymbol{g}(x) = 1+x + x3$

Messages	Code Vectors	Code polynomials
(0 0 0 0)	0 0 0 0 0 0 0	$\mathbf{0} = 0 \cdot \mathbf{g}(X)$
(1 0 0 0)	1 1 0 1 0 0 0	$1 + X + X^3 = 1 \cdot \mathbf{g}(X)$
(0 1 0 0)	0 1 1 0 1 0 0	$X + X^2 + X^4 = X \cdot \mathbf{g}(X)$
(1 1 0 0)	1 0 1 1 1 0 0	$1 + X^2 + X^3 + X^4 = (1 + X) \cdot \mathbf{g}(X)$
(0 0 1 0)	0 0 1 1 0 1 0	$X^2 + X^3 + X^5 = X^2 \cdot \mathbf{g}(X)$
(1 0 1 0)	1 1 1 0 0 1 0	$1 + X + X^2 + X^5 = (1 + X^2) \cdot \mathbf{g}(X)$
(0 1 1 0)	0 1 0 1 1 1 0	$X + X^3 + X^4 + X^5 = (X + X^2) \cdot \mathbf{g}(X)$
(1 1 1 0)	1 0 0 0 1 1 0	$1 + X^4 + X^5 = (1 + X + X^2) \cdot \mathbf{g}(X)$
(0 0 0 1)	0 0 0 1 1 0 1	$X^3 + X^4 + X^6 = X^3 \cdot \mathbf{g}(X)$
(1 0 0 1)	1 1 0 0 1 0 1	$1 + X + X^4 + X^6 = (1 + X^3) \cdot \mathbf{g}(X)$
(0 1 0 1)	0 1 1 1 0 0 1	$X + X^2 + X^3 + X^6 = (X + X^3) \cdot \mathbf{g}(X)$
(1 1 0 1)	1 0 1 0 0 0 1	$1 + X^2 + X^6 = (1 + X + X^3) \cdot \mathbf{g}(X)$
(0 0 1 1)	0 0 1 0 1 1 1	$X^2 + X^4 + X^5 + X^6 = (X^2 + X^3) \cdot \mathbf{g}(X)$
(1 0 1 1)	1 1 1 1 1 1 1	$1 + X + X^2 + X^3 + X^4 + X^5 + X^6 = (1 + X^2 + X^3) \cdot \mathbf{g}(X)$
(0 1 1 1)	0 1 0 0 0 1 1	$X + X^5 + X^6 = (X + X^2 + X^3) \cdot \mathbf{g}(X)$
(1 1 1 1)	1 0 0 1 0 1 1	$1 + X^3 + X^5 + X^6 = (1 + X + X^2 + X^3) \cdot \mathbf{g}(X)$

Encoding of Cyclic Codes

Encoding process: (1) Multiply $\boldsymbol{u}(x)$ by x^{n-k}; (2) divide $x^{n-k}\boldsymbol{u}(x)$ by $\boldsymbol{g}(x)$; (3) form the code word $\boldsymbol{b}(x) + x^{n-k}\boldsymbol{u}(x)$.

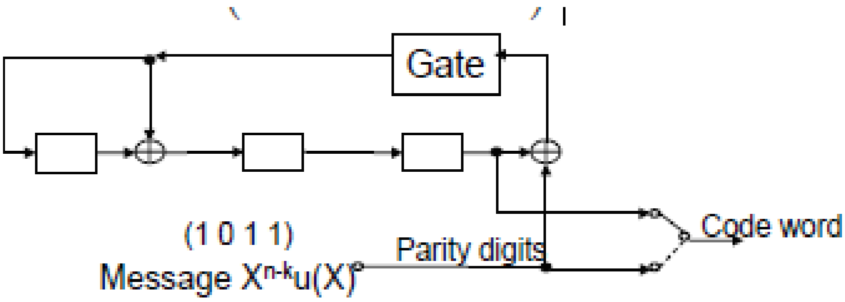


Example

• Consider the (7, 4) cyclic code generated by $\boldsymbol{g}(x) = 1+x + x^3$. Suppose that the message $\boldsymbol{u}$ = (1 0 1 1) is to be encoded. The contents in the register are as follows:

Input	Register contents
	0 0 0 (initial state)
1	1 1 0 (first shift)
1	1 0 1 (second shift)
0	1 0 0 (third shift)
1	1 0 0 (fourth shift

After four shifts, the contents of the register are (1 0 0). Thus the complete code vector is (1 0 0 1 0 1 1).



Encoding by Parity Polynomial

Since $hk = 1$, we have

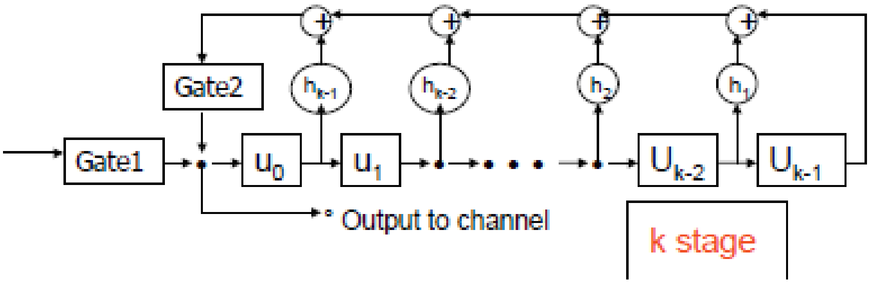
$V_{n-k-j} = \sum_{i=0}^{k-1} h\ i\ v\ n - i - j$ for $1 \leq j \leq n - k$,

which is known as a *difference equation*.

$v_{n-k-1} = h_0 v_{n-1} + h_1 v_{n-2} + \cdot \quad \cdot \quad \cdot + h_{k-1} v_{n-k} = u_{k-1} + h_1 u_{k-2} + \cdot \quad \cdot \quad \cdot + h_{k-1} u_0$

$v_{n-k-2} = u_{k-2} + h_1 u_{k-3} + \cdot \quad \cdot \quad \cdot \ + h_{k-1} v_{n-k-1}$

• Encoding circuit:



Syndrome Computation

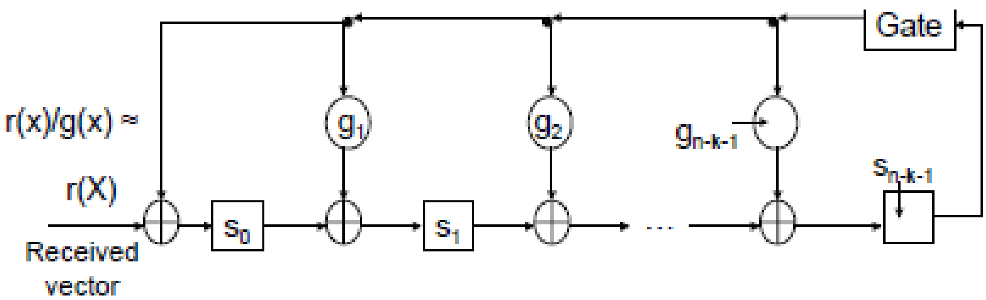
Let $\boldsymbol{r} = (r_0, r_1, \ldots, r_{n-1})$ be the received vector. The *syndrome* is calculated as $\boldsymbol{s} = \boldsymbol{r} \cdot \boldsymbol{H}^T$, where $\boldsymbol{H}$ is the parity-check matrix.

• If syndrome is not identical to zero, $\boldsymbol{r}$ is not a code vector and the presence of errors has been detected.

• Dividing $\boldsymbol{r}(x)$ by the generator polynomial $\boldsymbol{g}(x)$, we obtain

$\boldsymbol{r}(x) = \boldsymbol{a}(x)\boldsymbol{g}(x) + \boldsymbol{s}(x)$.

• The $n - k$ coefficients of $\boldsymbol{s}(x)$ form the syndrome $\boldsymbol{s}$. We call $\boldsymbol{s}(x)$ the *syndrome*.



If $\boldsymbol{C}$ is a systematic code, then the syndrome is simply the vector sum of the received parity digits and the parity-check digits recomputed from the received information digits.

• Let $\boldsymbol{s}(x)$ be the syndrome of a received polynomial $\boldsymbol{r}(x)$. Then the remainder $\boldsymbol{s}^{(1)}(x)$ resulting from dividing $x\boldsymbol{s}(x)$ by the generator polynomial $\boldsymbol{g}(x)$ is the syndrome of $\boldsymbol{r}^{(1)}(x)$, which is a
cyclic shift of $\boldsymbol{r}(x)$.

Decoding of Cyclic Codes

Decoding of linear codes consists of three steps: (1) syndrome computation; (2) association of the syndrome to an error pattern; (3) error correction.

• The cyclic structure structure of a cyclic code allows us to decode a received vector $\boldsymbol{r}(x)$ in serial manner.

• The received digits are decoded one at a time and each digit is decoded with the same circuitry.

• The decoding circuit checks whether the syndrome ß(x) corresponds to a correctable error pattern $\boldsymbol{e}(x)$ with an error at the highest-order position x^{n-1} (i.e., $e_{n-1} = 1$).

• If ß(x) does not correspond to an error pattern with $en-1 = 1$, the received polynomial and the syndrome register are cyclically shifted once simultaneously. By doing this, we have $\boldsymbol{r}^{(1)}(x)$ and $\boldsymbol{s}^{(1)}(x)$.

• The second digit r_{n-2} of $\boldsymbol{r}(x)$ becomes the first digit of $\boldsymbol{r}^{(1)}(x)$. The same decoding processes.

• If the syndrome $\boldsymbol{s}(x)$ of $\boldsymbol{r}(x)$ does correspond to an error pattern with an error at the location x^{n-1}, the first received digit r_{n-1} is an erroneous digit and it must be corrected by taking the sum

• This correction results in a modified received polynomial, denoted by

$\boldsymbol{r}_1(x) = r_0 + r_1x + \cdot \cdot \cdot + r_{n-2}x^{n-2} + (r_{n-1} \oplus e_{n-1})x^{n-1}$.

• The effect of the error digit $en-1$ on the syndrome can be achieved by adding the syndrome of $\boldsymbol{e}'(x) = x^{n-1}$ to $\boldsymbol{s}(x)$.

• The syndrome $\boldsymbol{s}_1^{(1)}$ of $\boldsymbol{r}_1^{(1)}(x)$ is the remainder resulting from dividing $x[\boldsymbol{s}(x) + x^{n-1}]$ by the generator polynomial $\boldsymbol{g}(x)$.

UNIT-III

Base Band Pulse Transmissions

In electronics and telecommunications, **pulse shaping** is the process of changing the waveform of transmitted pulses. Its purpose is to make the transmitted signal better suited to its purpose or the communication channel, typically by limiting the effective bandwidth of the transmission. By filtering the transmitted pulses this way, the intersymbol interference caused by the channel can be kept in control. In RF communication, pulse shaping is essential for making the signal fit in its frequency band.

Typically pulse shaping occurs after line coding and modulation.

Need for pulse shaping

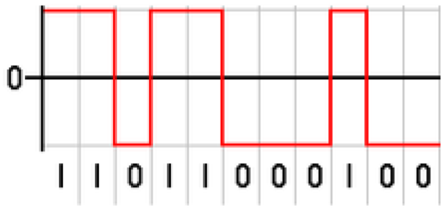
Transmitting a signal at high modulation rate through a band-limited channel can create intersymbol interference. As the modulation rate increases, the signal's bandwidth increases. When the signal's bandwidth becomes larger than the channel bandwidth, the channel starts to introduce distortion to the signal. This distortion usually manifests itself as intersymbol interference.

The signal's spectrum is determined by the pulse shaping filter used by the transmitter. Usually the transmitted symbols are represented as a time sequence of dirac delta pulses. This theoretical signal is then filtered with the pulse shaping filter, producing the transmitted signal. The spectrum of the transmission is thus determined by the filter.

In many base band communication systems the pulse shaping filter is implicitly a boxcar filter. Its Fourier transform is of the form *sin(x)/x*, and has significant signal power at frequencies higher than symbol rate. This is not a big problem when optical fibre or even twisted pair cable is used as the communication channel. However, in RF communications this would waste bandwidth, and only tightly specified frequency bands are used for single transmissions. In other words, the channel for the signal is band-limited. Therefore better filters have been developed, which attempt to minimise the bandwidth needed for a certain symbol rate

Pulse shaping filters

Not every filter can be used as a pulse shaping filter. The filter itself must not introduce intersymbol interference — it needs to satisfy certain criteria. The Nyquist ISI criterion is a commonly used criterion for evaluation, because it relates the frequency spectrum of the transmitter signal to intersymbol interference



A typical NRZ coded signal is implicitly filtered with a sinc filter.

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Examples of pulse shaping filters that are commonly found in communication systems are:

- Sinc shaped filter
- Raised-cosine filter
- Gaussian filter

Sender side pulse shaping is often combined with a receiver side matched filter to achieve optimum tolerance for noise in the system. In this case the pulse shaping is equally distributed between the sender and receiver filters. The filters' amplitude responses are thus pointwise square roots of the system filters.

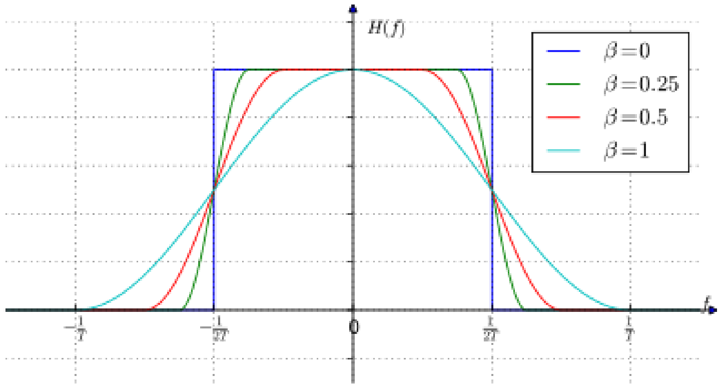
Other approaches that eliminate complex pulse shaping filters have been invented. In OFDM, the carriers are modulated so slowly that each carrier is virtually unaffected by the bandwidth limitation of the channel.

Sinc filter

It is also called as Boxcar filter as its frequency domain equivalent is a rectangular shape. Theoretically the best pulse shaping filter would be the sinc filter, but it cannot be implemented precisely. It is a non-causal filter with relatively slowly decaying tails. It is also problematic from a synchronisation point of view as any phase error results in steeply increasing intersymbol interference.

Raised-cosine filter

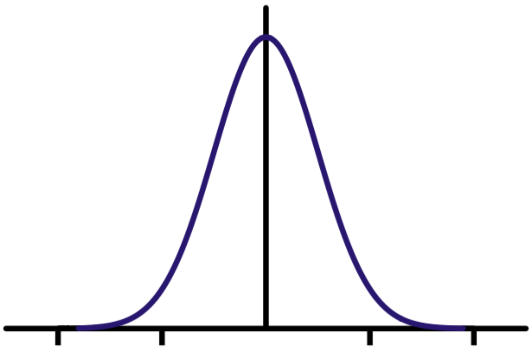
The **raised-cosine filter** is a filter frequently used for pulse-shaping in digital modulation due to its ability to minimise intersymbol interference (ISI). Its name stems from the fact that the non-zero portion of the frequency spectrum of its simplest form (β=1) is a cosine function, raised up to sit above the *f* (horizontal) axis .Raised-cosine filters are practical to implement and they are in wide use. They have a configurable excess bandwidth, so communication systems can choose a trade off between simpler filter and spectral efficiency.



Amplitude response of raised-cosine filter with various roll-off factors

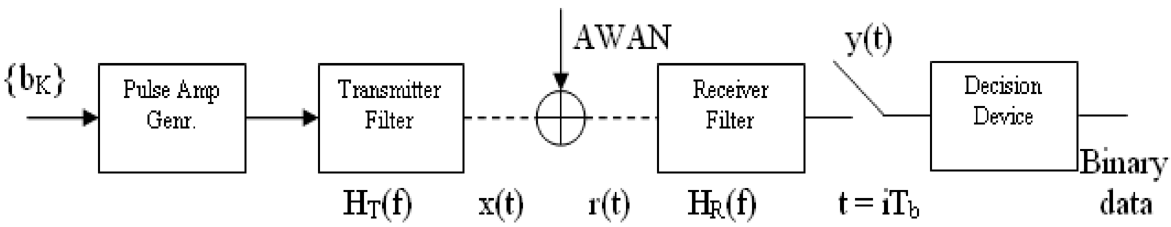
Gaussian filter

In electronics and signal processing, a **Gaussian filter** is a filter whose impulse response is a Gaussian function (or an approximation to it). This behavior is closely connected to the fact that the Gaussian filter has the minimum possible group delay. It is considered the ideal time domain filter, just as the sinc is the ideal frequency domain filter.[1]hese properties are important in areas such as oscilloscopes[2] and digital telecommunication systems.[3]



Shape of the impulse response of a typical Gaussian filter

A Baseband Signal Receiver



The above receiver is known as OPTIMUM / MATCHED receiver.

→ Let us consider very simple & basic circuit for the reception of digital signal (detection of binary signal) over a sequence of electronic pulses with $\pm$ AV of amplitude with duration "T" where "T" is bit duration.

→ The transmitted binary signal is corrupted by noise during its travel through the channel. The received signal**:**

r(t) = x(t) + n(t) ← Noise. (AWGN with $S_n(f) = \frac{No}{2} = \eta/2$)

desired signal

Additive White Gaussian Noise.

→ The i/P to the receiver filter is x(t) + n(t).

Calculation of SNR at the o/p of Receiver filter:

The O/P of an integrator can be written as

$r(t) = \frac{1}{RC}\int_{o}^{T} y(t)dt$

y(t) = x(t) + n(t)

$r(t) = \frac{1}{RC}\int_{o}^{T} x(t) + n(t)\ dt$

$= \frac{1}{RC}\int_{o}^{T} x(t)dt + \frac{1}{RC}\int_{o}^{T} n(t)\ dt$

$= \frac{1}{\tau}\int_{o}^{T} x(t)dt + \frac{1}{\tau}\int_{o}^{T} n(t)dt$

$= x_0(t) + n_0(t)$ → (1)

$x_0(t)$ = o/p signal

$n_0(t)$ = o/p Noise

→ $x_0(t) = \frac{1}{\tau}\int_{o}^{T} x(t)\ dt$

$x_0(t) \pm \frac{AT}{\tau}$ → (2)

Where τ is time constant of an integrator

→ The normalized o/p signal power(R = 1Ω)

$S_0\ (t) = \frac{\overline{x_0^2(t)}}{R} = \overline{x_0^2(t)}$

$S_0(t)$ = o/p signal power

$= \frac{A^2\ T^2}{\tau^2}$ → (3)

→ Calculate the total power of the signal.

X(t) = 10 + 5 Sin 1000πt

$= 100 + \frac{25}{2}$

=100 + 12. 5 = 112.5 W

→ Calculate of o/p Noise power

1) Before calculation of o/p noise power let us recall network that performs integration operation.

2) Integrator has a TF $H(f) = \frac{1-e^{-j\omega T}}{j\omega RC}$

Where T = T_b

Note:

Note:

Integrator performs integration operation on i/p signal over bit duration T_b.

→ Substituting $\omega = 2\pi f$

Rc = , the above equation takes the following form.

$H(f) = \frac{1-e^{-j2\pi fT}}{j2\pi f\tau}$

→simplifying the obove equation using Euler's identity. We get

$= \frac{1-Cos\ (2\pi fT)+\ j.Sin\ (2\pi fT)}{j2\pi f\tau}$ → (4)

→ On separating real & imaginary parts.

$H(f) = \frac{Sin\ (2\pi fT)}{2\pi f\tau} + \left(\frac{Cos\ (2\pi fT)}{2\pi f\tau}\right)$ j

→ Magnitude of H(f)

$$|H(f)| = \sqrt{\frac{Sin^2(2\pi fT)}{(2\pi f\tau)^2} + \frac{Cos^2(2\pi f\tau) + 1 - 2Cos\,(2\pi f\tau)}{(2\pi f\tau)^2}}$$

$$= \sqrt{\frac{2(1 - Cos2\pi fT)}{4(\pi f\tau)}} = \frac{Sin^2(2\pi fT)}{\pi f\tau}$$

$$|H(f)|^2 = \frac{Sin^2(\pi fT)}{(\pi f\tau)^2} \to (5)$$

→The average power of output noise " n_0 (t)" is equals to area under power spectral density curve.

$$n_0(t) = \int_{-\infty}^{\infty} S_{n_0}(f)\, df$$

S_{n_0}(f) = o/p PSD of o/p noise n_0(t)

$$= \int_{-\infty}^{\infty} S\,(f)\, df$$

→ o/p Noise power = $\frac{\overline{n_0^2(t)}}{R}$

→ We Know that i/P & o/p PSD'S of base band signal receiver are related by the following equation.

$$\boxed{S_{n_0}(f) = |H(f)|^2 S_{n_i}(f)}$$

$$n_0(t) = \int_{-\infty}^{\infty} \frac{Sin^2(\pi fT)}{(\pi f\tau)^2} \cdot \frac{\eta}{2}\; df \to (7)$$

$$= \int_0^T \frac{1 + Cos\,2\pi fT}{2(\pi f\tau)^2} \cdot \frac{\eta}{2}\; df$$

$$= \frac{\eta}{4} \int_0^T \left(\frac{1}{(\pi f\tau)^2} + \frac{Cos\,2\pi fT}{(\pi f\tau)^2}\right) df$$

$$= \frac{\eta}{4} \pi^2\tau^2 \left[\frac{-1}{f}\right]_0^T + \frac{\eta}{4\pi^2\tau^2} \int_0^T \frac{Cos(\,2\pi fT\,)}{f^2}\; df$$

$$= \frac{\eta}{4\pi^2\tau^2}$$

Now let $\pi f\tau = x$ $\qquad f = \frac{x}{\pi\tau}$

$dx = \pi\tau\, df$ $\qquad \pi fT = \frac{\pi\tau}{\tau}$

$df = \frac{1}{\pi\tau} dx$

$$n_0(t) = \frac{\eta}{2} \int_{-\infty}^{\infty} \frac{\sin^2\left(\frac{x}{\pi\tau}\right)}{x^2} \cdot \frac{1}{\pi\tau}\, dx$$

Rearranging the above equation

$$= \frac{\eta}{2} \int_{-\infty}^{\infty} \frac{\sin^2\left(\frac{xT}{\tau}\right)}{\left(\frac{xT}{\tau}\right)^2} \cdot \left(\frac{T}{\tau}\frac{T}{\tau}\right)^2 \cdot \frac{1}{\pi\tau}\, dx$$

Now let $\frac{\pi\tau}{\tau} = u$

$dx = \frac{T}{\tau} du$

$$= \frac{\eta}{2} \int_{-\infty}^{\infty} \frac{\sin^2 u}{u^2} \left(\frac{u}{x}\right)^2 \cdot \frac{1}{\pi\tau} \frac{T}{\tau}\, du$$

$$= \frac{\eta}{2} \int_{-\infty}^{\infty} \frac{\sin^2 u}{u^2} \cdot \frac{T}{\pi\tau^2}\, du$$

$$= \frac{\eta T}{2\pi\tau^2} \int_{-\infty}^{\infty} \frac{\sin^2 u}{u^2} \cdot du \to (7)$$

∴ $\frac{Sin\,u}{u}$ Squared the above equation can be written as

$$n_0(t) = \frac{\eta T}{2\pi\tau^2}\, 2 \int_0^{\infty} \frac{\sin^2 u}{u^2} \cdot du$$

$$= \frac{\eta T}{2\pi\tau^2} \times \frac{2\pi}{2}$$

$$\boxed{n_0(t) = \frac{\eta T}{2\tau^2}} \to (8)$$

$\rightarrow$ SNR $= \frac{S_0}{N_0} = \frac{Average\ pwr\ of \frac{o}{p}\ signal}{Averge\ pwr\ of \frac{o}{p} noise}$

$= \frac{\frac{A^2T^2}{\tau^2}}{\frac{\eta T}{2\tau^2}}$

$= \frac{A^2}{\frac{\eta}{2}}$

$$\boxed{SNR = \frac{2A^2T}{\eta}}$$

$\rightarrow$ o/p SNR can be maximized by increasing the amplitude of signal.
$\rightarrow$o/p SNR is also known as "figure of merit" (FOM).
$\rightarrow$ Show that for an integrate & dump filter receiver, the maximum SNR is expressed as.

$\left(\frac{S}{N}\right)_0 = \frac{2E}{N_0}$

Given that i/p signal x(t) is rectangular pulses of amplitudes $\pm$ A & duration T.

Power of x(t) = $A^2 = \frac{E_b}{T_b}$

$$A = \sqrt{\frac{E_b}{T_b}}$$

$$\therefore \left(\frac{S}{N}\right)_0 = \frac{2A^2T}{\eta}$$

$$= \frac{2\frac{E_b}{T}T}{\eta}$$

$$\left(\frac{S}{N}\right)_0 = \frac{2E}{\eta}$$

Calculation of probability of bit error (error probability) for a baseband signal receiver

Noise effects the transmitted signal doing its travel through the channel.
$\rightarrow$ Effect of noise on transmitted signal may lead to wrong decision at the receiver end.
$\rightarrow$ The transmitted signal x(t) is a binary wave form.

x(t) = $\pm$ A $\qquad$ $0 \le t \le T$ ($T = T_b$ Bit interval / Bit duration)

The o/p of an integrator can be expressed as

$$r(t) = \frac{1}{RC}\int_O^T x(t)dt + \frac{1}{RC}\int_O^T n(t)\,dt$$

$= x_0(t) + n_0(t)$

$x_0(t)$ = o/p signal

$n_0(t)$ = o/p Noise

$\rightarrow$ For a positive pulse

x(t) = +AV

$\rightarrow x_0(t) = \frac{AT}{\tau}$

$\rightarrow$ For a negative pulse

x(t) = - AV

$\rightarrow x_0(t) = \frac{-AT}{\tau}$

Therefore, the o/p of an integrator can be expressed as

$r(t) = x_0(t) + n_0(t)$

$$r(t) = \frac{AT}{\tau} + n_0(t)$$

$$r(t) = \frac{-AT}{\tau} + n_0(t)$$

$\rightarrow$ Let us consider Bit "0" is transmitted, then the output of an integrator.

$$r(t) = \frac{-AT}{\tau} + n_0(t)$$

$\rightarrow$ Let us assume $n_0(t) > \frac{AT}{\tau}$

Then r(t) → +Ve
And decision is in the favour of "Bit 1"
(wrong Decision)
→Error probability

$P_e = \left[n_0(t) > \frac{AT}{\tau}\right]$

P_e = probability of bit error / error probability/ probability of symbol error.
→ Let us consider the transmitted bit is "1" then o/p of integrator.

$$r(t) = \frac{AT}{\tau} + n_0(t)$$

and further $n_0(t) > \frac{-AT}{\tau}$
Then r(t) → -Ve
And decision is in favour of "Bit 0"
(wrong decision)
→ Error probability

$P_e = \left[n_0(t) > \frac{AT}{\tau}\right]$

→ Let us recall the probability density function of Gaussian random variable x.

$$f_x(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}}\, e^{-\{x-\mu_x\}^2/2\sigma_x^2}$$

μ_x → Mean value of Gaussian Random variable 'x'
σ_x^2 → variance of Gaussian Random variable 'x'
$\sigma_x^2 = E[x^2] - \{E[x]\}^2$

$= \overline{x^2} - [\bar{x}]^2$

→Let us consider $n_0(t)$ is a additive white Gaussian noise with zero mean.
Then the pdf of additive white Gaussian Noise.
$n_0(t)$ is given as

$$f_x(n_0(t)) = \frac{1}{\sqrt{2\pi\sigma_x^2}} \cdot e^{-\{n_0(t)\}^2/2\sigma_x^2}$$

→ var [$n_0(t)$] = $\overline{n_0^2(t)} - (0)^2$

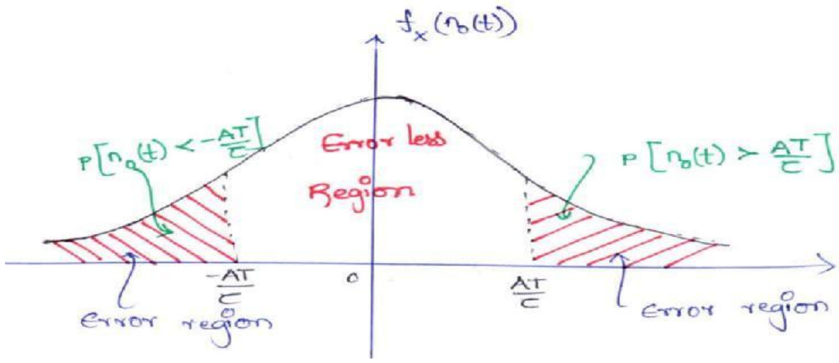
$= \overline{n_0^2(t)}$

$\overline{n_0^2(t)}$ = Mean sq. value of o/p noise $n_0(t)$

$= \frac{N_0 T}{2\tau}$

$$F_x(n_0(t)) = \frac{1}{\sqrt{2\pi\frac{\eta T}{2\tau^2}}} \cdot e^{-\frac{\{n_0(t)\}^2}{2\cdot\frac{\eta T}{2\tau^2}}}$$

$$\boxed{F_x(n_0(t)) = \frac{\tau}{\sqrt{\pi\eta T}} \cdot e^{\frac{-\{n_0(t)\}^2}{2\eta T}}}$$



Pdf of Gaussian noise $n_0(t)$

→ Since, the pdf curve is symmetric, therefore we can write probability of error.

$$P_e = p[\, n_0(t) > \frac{AT}{\tau}] = p[\, n_0(t)\ \frac{-AT}{\tau}]$$

→From the properties of pdf

$$P[\, n_0(t) > \frac{AT}{\tau}] = \int_{AT/\tau}^{\alpha} f_x(n_0(t)\ d(n_0(t))$$

$$\{\therefore p[x \le x] = \int_{-\infty}^{x} f_x(x)dx\}$$

$$= \int_{AT/\tau}^{\alpha} \frac{\tau}{\sqrt{\pi\eta T}}\, e^{\frac{-\{n_0(t)\}^2}{2\eta T/\tau^2}} \quad d(n_0(t))$$

$$= \frac{\tau}{\sqrt{\pi\eta T}} \int_{AT/\tau}^{\alpha} e^{\frac{-\{n_0(t)\}^2}{2\eta T/\tau^2}}\, d(n_0(t))$$

→ In the above equation let us substitute

$$\frac{\{n_0(t)\}^2}{N_0 T/\tau^2} = y^2$$

$$\frac{\cancel{2}n_0(t)\ d(n_0(t))}{N_0 T/\tau^2} = \cancel{2}y\ dy$$

$$n_0(t)\ \boxed{d(n_0(t)) = \sqrt{\frac{N_0 T}{\tau^2}}\ dy}$$

$$n_0(t) = \sqrt{\frac{N_0 T}{\tau^2}}\ y$$

$$y = \frac{AT}{\sqrt{N_0 T}} = \sqrt{\frac{A^2 T}{N_0}}$$

$$p_e = \int_{\sqrt{\frac{A^2T}{N_0}}}^{\infty} \frac{1}{\sqrt{x}} . e^{-y^2}\ dy$$

$$= \frac{1}{\sqrt{x}} \int_{\sqrt{\frac{A^2T}{N_0}}}^{\infty} . e^{-y^2}\ dy$$

→ There are two special functions

Erf (u) . erf_c & Q(u)

Complementary error function

That oftenly comes in the soulution of "Theory of error & probability theory"

$$\mathrm{erf}_c(u) = \frac{2}{\sqrt{x}} \int_u^{\infty} e^{-y^2}\ dy$$

$$p_e = p\,[\, n_0(t) > \frac{AT}{\tau}] = \frac{1}{2}\,.\mathrm{erf}_c\left(\sqrt{\frac{A^2T}{N_0}}\right)$$

→ error function is are monotonically decreasing functions.

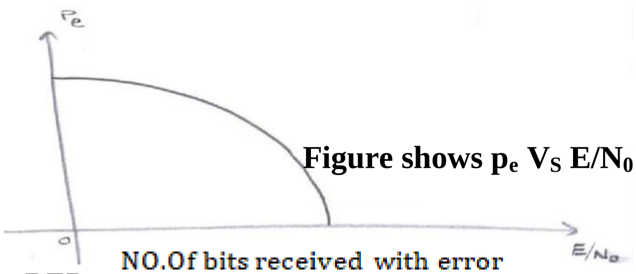


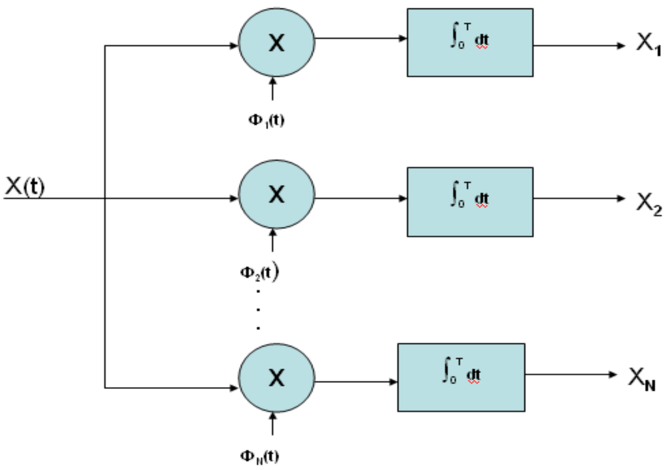
Figure shows p_e V_S E/N_0

Bit error rate:

Ideally BER = 10^{-12}

$$BER = \frac{\text{NO.Of bits received with error}}{\text{No.of bits transmitted}}$$

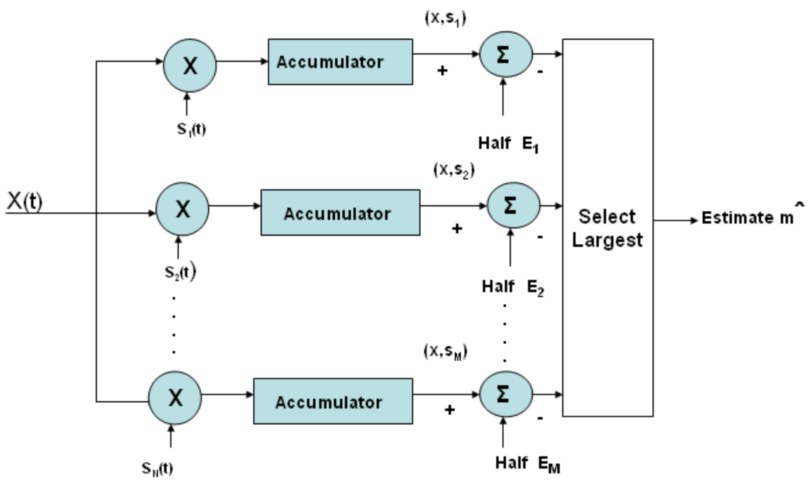
Correlative receiver



Detector

For an AWGN channel and for the case when the transmitted signals are equally likely, the optimum receiver consists of two subsystems 1) .Receiver consists of a bank of M product-integrator or correlators $\Phi 1(t)$,$\Phi 2(t)$ …….$\Phi_M(t)$ orthonormal function.

The bank of correlator operate on the received signal x(t) to produce observation vector x



Vector Receiver

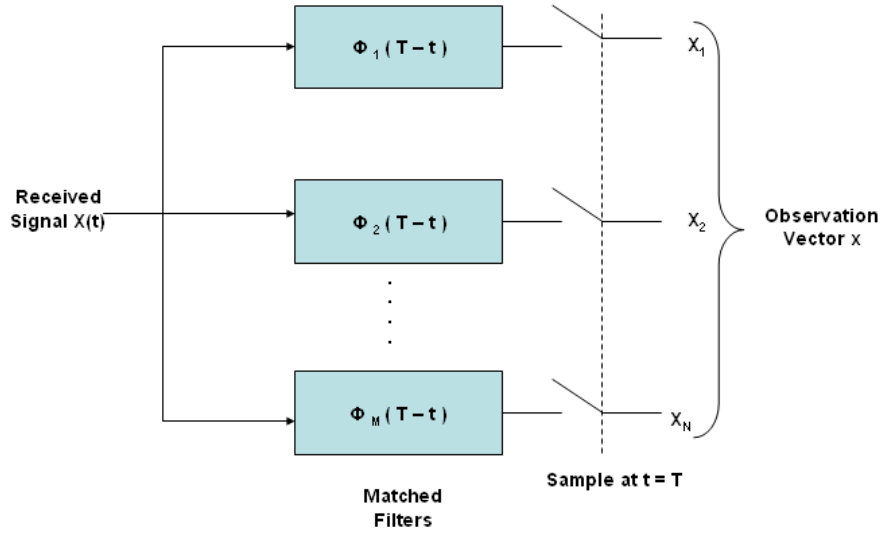
Implemented in the form of maximum likelihood detector that operates on observation vector **x** to produce an estimate of the transmitted symbol m_i i = 1 to M, in a way that would minimize the average probability of symbol error. The N elements of the observation vector **x** are first multiplied by the corresponding N elements of each of the M signal vectors s_1, s_2… s_M , and the resulting products are successively summed in accumulator to form the corresponding set of Inner products {(x, s_k)} k= 1, 2 ..M. The inner products are corrected for the fact that the transmitted signal energies may be unequal. Finally, the largest in the resulting set of numbers is selected and a corresponding decision on the transmitted message made. The optimum receiver is commonly referred as a **correlation receiver**

MATCHED FILTER

Science each of the orthonormal basic functions are $\Phi_1(t)$,$\Phi_2(t)$ …….$\Phi_M(t)$ is assumed to be zero outside the interval $0<t<T$. we can design a linear filter with impulse response $h_j(t)$, with the received signal x(t) the fitter output is given by the convolution integral $y_j(t) = x_j$ where x_j is the j th correlator output produced by the received signal x(t). A filter whose

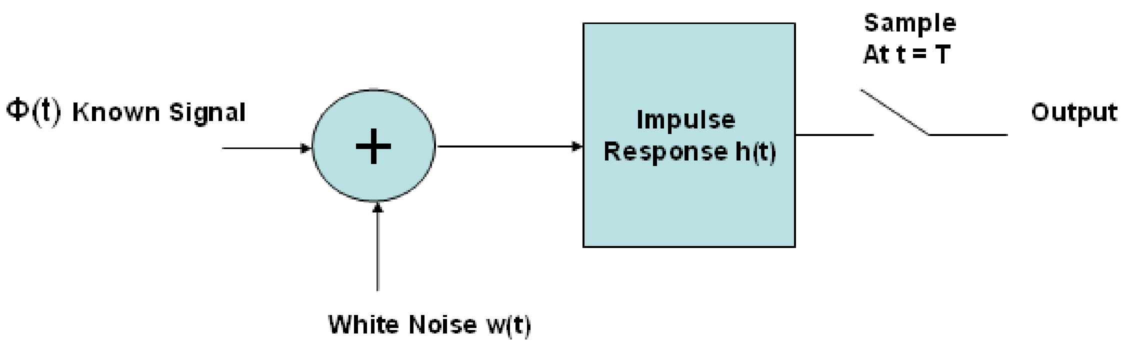
impulse response is time-reversed and delayed version of the input signal is said to be matched to . correspondingly , the optimum receiver based on this is referred as the **matched filter receiver**. For a matched filter operating in real time to be physically realizable, it must be causal.

For causal system



Detector Part Of Matched Filter Receiver

MATCHED FILTER



Φ(t) = input signal
h(t) = impulse response
W(t) =white noise
The impulse response of the matched filter is time-reversed and delayed version of the input signal.

MATCHED FILTER PROPERTIES

PROPERTY 1: The spectrum of the output signal of a matched filter with the matched signal as input is, except for a time delay factor, proportional to the energy spectral density of the input signal.

PROPERTY 2: The output signal of a Matched Filter is proportional to a shifted version of the autocorrelation function of the input signal to which the filter is matched.

PROPERTY 3: The output Signal to Noise Ratio of a Matched filter depends only on the ratio of the signal energy to the power spectral density of the white noise at the filter input.

PROPERTY 4 The Matched Filtering operation may be separated into two matching conditions; namely spectral phase matching that produces the desired output peak at time T, and the spectral amplitude matching that gives this peak value its optimum signal to noise density ratio.

MAXIMUM LIKELIHOOD DETECTOR:

Detection of known signals in noise

Assume that in each time slot of duration T seconds, one of the M possible signals $S_1(t)$, $S_2(t)$. . . . . . . $S_M(t)$ is transmitted with equal probability of 1/M. Then for an AWGN channel a possible realization of sample function x(t) of the received random process X(t) . where w(t) is sample function of the white Gaussian noise process W(t), with zero mean and PSD N0/2. The receiver has to observe the signal x(t) and make a **best estimate** of the transmitted signal si(t) or equivalently symbol mi The transmitted signal $S_i(t)$, i= 1to M , is applied to a bank of correlators, with a common input and supplied with an appropriate set of N orthonormal basic functions, the resulting correlator outputs define the signal vector **Si.** knowing **Si** is as good as knowing the transmitted signal Si(t) itself, and vice versa. We may represents si(t) by a point in a Euclidean space of dimensions N ≤ M. . Such a point is referred as transmitted signal point or message point. The collection of M message points in the N Euclidean space is called a **signal constellation.**

When the received signal x(t) is applied to the bank o N correlators , the output of the correlator define a new vector **x** called observation vector. this vector **x** differs from the signal vector s_i by a random noise vector **w.**The vectors **x** and **w** are sampled values of the random vectors **X** and **W** respectively. the noise vector **w** represents that portion of the noise w(t) which will interfere with the detected process. Based on the observation vector **x**, we represent the received signal s(t)by a point in the same Euclidean space, we refer this point as **received signal point**.

In the detection problem , the observation vector x is given, we have to perform a mapping from **x** to an estimate of the transmitted symbol, in away that would minimize the average probability of symbol error in the decision. The **maximum likelihood detector** provides solution to this problem.

Optimum transmitter & receiver

- Probability of errordepends on signal to noise ratio
- As the SNR increases the probability of error decreases
- An optimum transmitter and receiver is one which maximize the SNR and minimize the probability of error.

Inter symbol Interference

Generally, digital data is represented by electrical pulse, communication channel is always band limited. Such a channel disperses or spreads a pulse carrying digitized samples passing through it. When the channel bandwidth is greater than bandwidth of pulse, spreading of pulse is very less. But when channel bandwidth is close to signal bandwidth, i.e. if we transmit digital data which demands more bandwidth which exceeds channel bandwidth, spreading will occur and cause signal pulses to overlap. This overlapping is called **Inter Symbol Interference**. In short it is called ISI. Similar to interference caused by other sources, ISI causes degradations of signal if left uncontrolled. This problem of ISI exists strongly in Telephone channels like coaxial cables and optical fibers. In this chapter main objective is to study the effect of ISI, when digital data is transmitted through band limited channel and solution to overcome the degradation of waveform by properly shaping pulse.

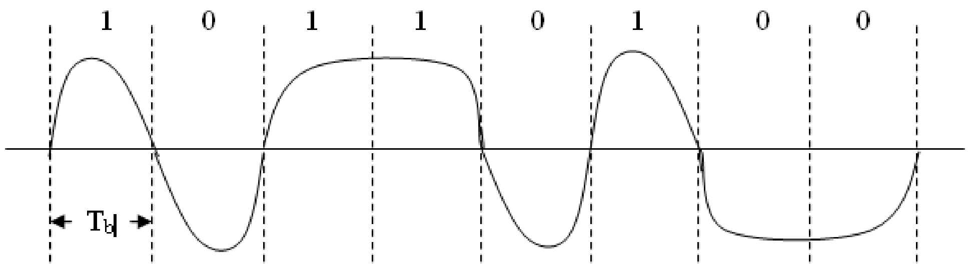
The effect of sequence of pulses transmitted through channel is shown in fig. The Spreading of pulse is greater than symbol duration, as a result adjacent pulses interfere. i.e. pulses get completely smeared, tail of smeared pulse enter into adjacent symbol intervals making it difficult to decide actual transmitted pulse. First let us have look at different formats of transmitting digital data.In base band transmission best way is to map digits or symbols into pulse waveform. This waveform is generally termed as **Line codes**.

EYE PATTERN

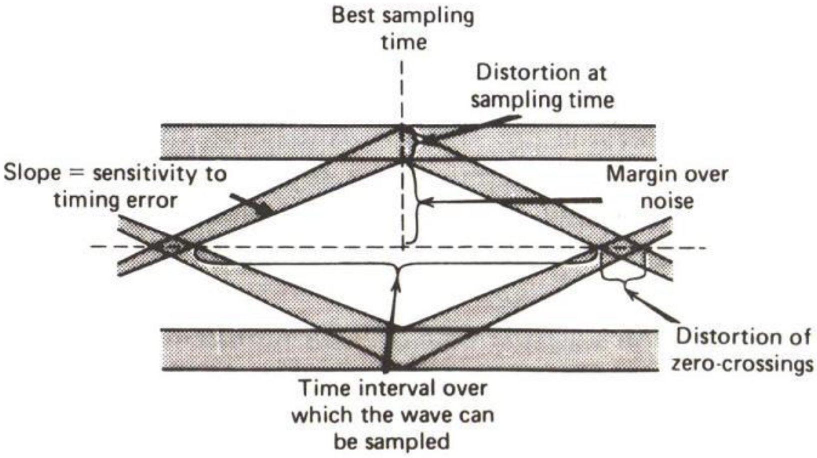
The quality of digital transmission systems are evaluated using the bit error rate. Degradation of quality occurs in each process modulation, transmission, and detection. The eye pattern is experimental method that contains all the information concerning the degradation of quality. Therefore, careful analysis of the eye pattern is important in analyzing the degradation mechanism.

• Eye patterns can be observed using an oscilloscope. The received wave is applied to the vertical deflection plates of an oscilloscope and the sawtooth wave at a rate equal to transmitted symbol rate is applied to the horizontal deflection plates, resulting display is eye pattern as it resembles human eye.

• The interior region of eye pattern is called eye opening



We get superposition of successive symbol intervals to produce eye pattern as shown below.



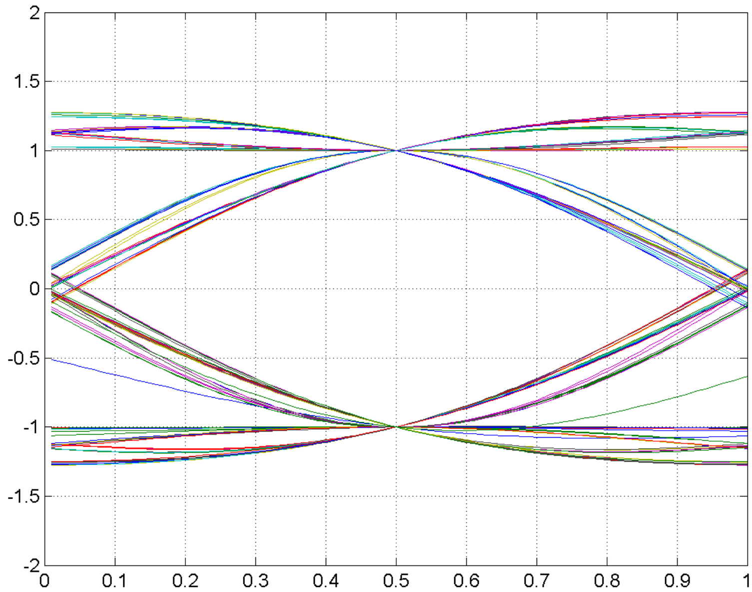
Interpretation of eye pattern

- The width of the eye opening defines the time interval over which the received wave can be sampled without error from ISI
- The optimum sampling time corresponds to the maximum eye opening
- The height of the eye opening at a specified sampling time is a measure of the margin over channel noise.

The sensitivity of the system to timing error is determined by the rate of closure of the eye as the sampling time is varied. Any non linear transmission distortion would reveal itself in an asymmetric or squinted eye. When the effected of ISI is excessive, traces from the upper portion of the eye pattern cross traces from lower portion with the result that the eye is completely closed.

Example of eye pattern:

Binary-PAM Perfect channel (no noise and no ISI)



Example of eye pattern: Binary-PAM with noise no ISI

EQUALISING FILTER

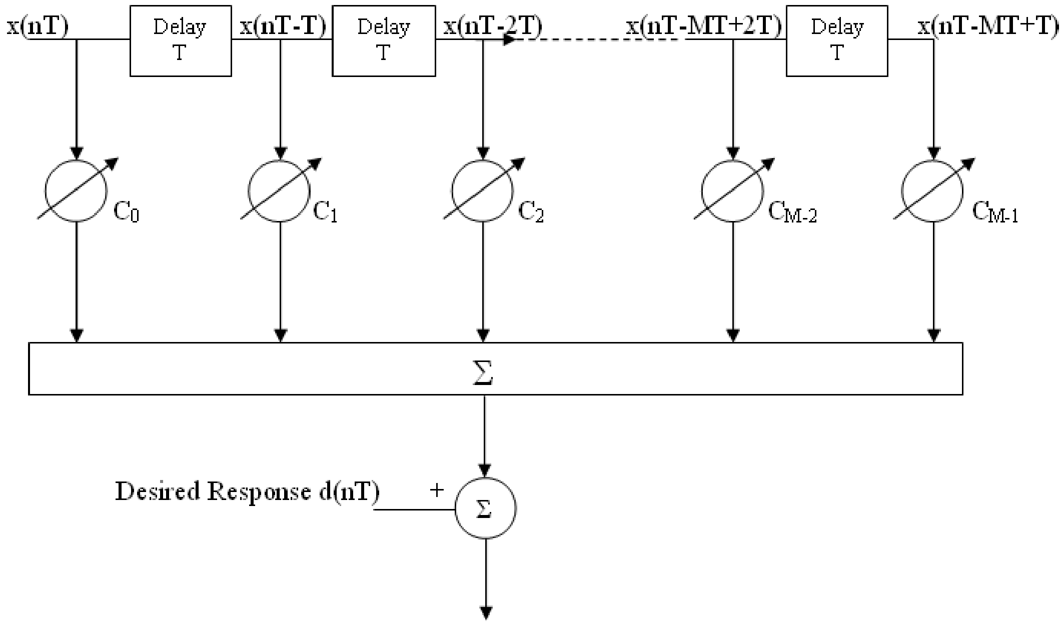
Adaptive equalization

• An equalizer is a filter that compensates for the dispersion effects of a channel. Adaptive equalizer can adjust its coefficients continuously during the transmission of data.

Pre channel equalization

- requires feed back channel
- causes burden on transmission.

Post channel equalization Achieved prior to data transmission by training the filter with the guidance of a training sequence transmitted through the channel so as to adjust the filter parameters to optimum values. **Adaptive equalization** – It consists of tapped delay line filter with set of delay elements, set of adjustable multipliers connected to the delay line taps and a summer for adding multiplier outputs.



The output of the Adaptive equalizer is given by

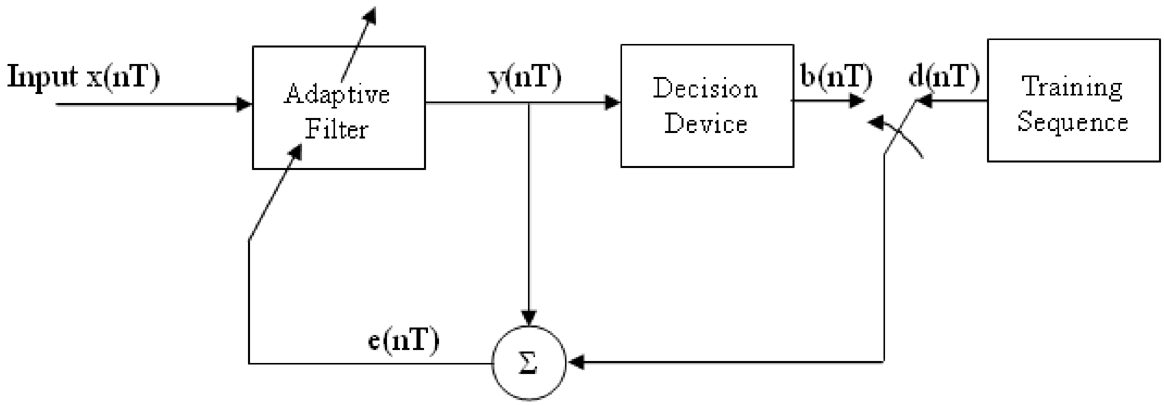
Y(nt)=ΣCi x(nT-iT)

Ci is weight of the ith tap Total number of taps are M .Tap spacing is equal to symbol duration T of transmitted signal In a conventional FIR filter the tap weights are constant and particular designed response is obtained. In the adaptive equaliser the **Ci**'s are variable and are adjusted by an algorithm

Two modes of operation

1. Training mode 2 . Decision directed mode

Mechanism of adaptation



Training mode A known sequence d(nT) is transmitted and synchronized version of it is generated in the receiver applied to adaptive equalizer.

This training sequence has maximal length PN Sequence, because it has large average power and large SNR, resulting response sequence (Impulse) is observed by measuring the filter outputs at the sampling instants.

The difference between resulting response y(nT) and desired response d(nT)is error signal which is used to estimate the direction in which the coefficients of filter are to be optimized using algorithms.

BANDPASS SIGNAL TRANSMISSION AND RECEPTION

Memoryless modulation techniques Modulation is defined as the process by which some characteristics of a carrier is varied in accordance with a modulating wave. In digital communications, the modulating wave consists of binary data or an M-ary encoded version of it and the carrier is sinusoidal wave. Different Shift keying methods that are used in digital modulation techniques are

- **Amplitude shift keying [ASK]**
- **Frequency shift keying [FSK]**
- **Phase shift keying [PSK]**

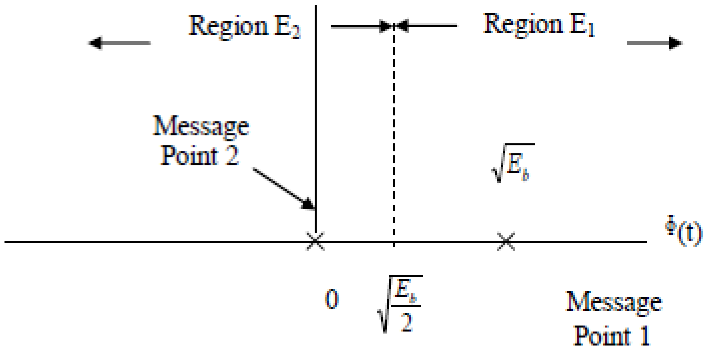
Amplitude Shift Keying (ASK) Modulation:

Amplitude shift keying (ASK) is a simple and elementary form of digital modulation in which the amplitude of a carrier sinusoid is modified in a discrete manner depending on the value of a modulating symbol. Let a group of 'm' bits make one symbol. Hence one can design $M = 2^m$ different baseband signals, $d_m(t)$, $0 \le m \le M$ and $0 \le t \le T$. When one of these symbols modulates the carrier, say, $c(t) = \cos\omega_c t$, the modulated waveform is:

$$s_m(t) = d_m(t).\cos\omega_c t \qquad 5.23.1$$

This is a narrowband modulation scheme and we assume that a large number of carrier cycles are sent within a symbol interval, i.e. $\frac{T}{\left(2\pi/\varpi_c\right)}$ is a large integer. It is obvious that the information is embedded only in the peak amplitude of the modulated signal. So, this is a kind of digital amplitude modulation technique. From another angle, one can describe this scheme of modulation as a one-dimensional modulation scheme where one basis function $\varphi_1(t) = \sqrt{\frac{2}{T}}.\cos\varpi_c t$, defined over $0 \le t \le T$ and having unit energy is used and all the baseband signals are linearly dependent.

The BASK system has one dimensional signal space with two messages (N=1, M=2)



Signal Space representation of BASK signal

In transmitter the binary data sequence is given to an on-off encoder. Which gives an output $\sqrt{Eb}$ volts for symbol 1 and 0 volt for symbol 0. The resulting binary wave [in unipolar form] and sinusoidal carrier are applied to a product modulator. The desired BASK wave is obtained at the modulator output.

In demodulator, the received noisy BASK signal x(t) is apply to correlator with coherent reference signal. The correlator output x is compared with threshold λ. If $x > \lambda$ the receiver decides in favour of symbol 1. If $x < \lambda$ the receiver decides in favour of symbol 0.

Frequency Shift Keying Modulation

Frequency Shift Keying (FSK) modulation is a popular form of digital modulation used in low-cost applications for transmitting data at moderate or low rate over wired as well as wireless channels. In general, an M-ary FSK modulation scheme is a power efficient modulation scheme and several forms of M-ary FSK modulation are becoming popular for spread spectrum communications and other wireless applications. In this lesson, our discussion will be limited to binary frequency shift keying (BFSK).

Binary FSK system has 2 dimensional signal space with two messages S1(t) and S2(t), [N=2 , m=2] they are represented

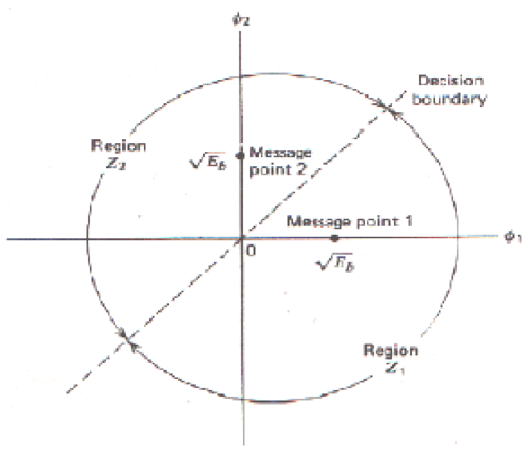


Fig. Signal Space diagram of Coherent binary FSK system.

PHASE SHIFT KEYING(PSK): In a Coherent binary PSK system the pair of signals $S_1(t)$ and $S_2(t)$ are used to represent binary symbol „1" and „0" respectively.

The signal space representation is as shown in fig (N=1 & M=2)

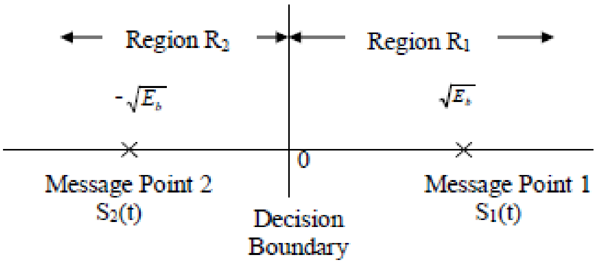


Fig. Signal Space Representation of BPSK

UNIT – IV
Digital Modulation Techniques

Introduction:

This unit is devoted to a study of digital modulation techniques and their merits, limitations & applications.

TYPES OF DIGITAL MODULATION:

1) Binary digital modulation techniques

a) ASK
b) FSK
c) PSK

2) M-ary digital modulation schemes

a) M-ary ASK
b) M-ary FSK
c) M – ary PSK

→ In digital modulation, digital modulating signal modulates analog carrier. Hence "Digital Modulation".

→Where as in analog modulation, analog modulating signal modulates analog carrier. Hence "analog modulation" .

→ Modulation is a process in which any one characteristics of carrier signal is varied in accordance to amplitude of modulating signal.

→ In digital communication, modulating wave consists of binary data.

→ $C(t) = A \cos(2\pi ft + \emptyset)$

Amplitude
Frequency
Phase.

Transmission of digital signals:

1) **Baseband data transmission:**

→ The digital data is transmitted over the channel directly. There is no carrier or any modulation. This is suitable for transmission over short distances.

2) Band Pass Data Transmission:

→ The digital data modulator high freq carrier.

→when it is required to transmit digital data over a band pass channel it is necessary to modulate the incoming data onto a carrier wave.

→ **Digital Modulations**

1) ASK
2) FSK
3) PSK

→ In ASK, amplitude of carrier is switched from one value to another value depending on modulating digital i/p

→ In PSK, phase of carrier is switched from one value to another value depending on modulating digital i/p.

→ in FSK frequency of carrier is switched from one value to another value depending on modulating digital i/p.

Digital modulation techniques classified in to two types depending on whether receiver is equipped with phase recovery circuit or not.

1) Coherent Digital Modulation Techniques
2) Non Coherent Digital Modulation Techniques

1) Coherent Digital Modulation Techniques:

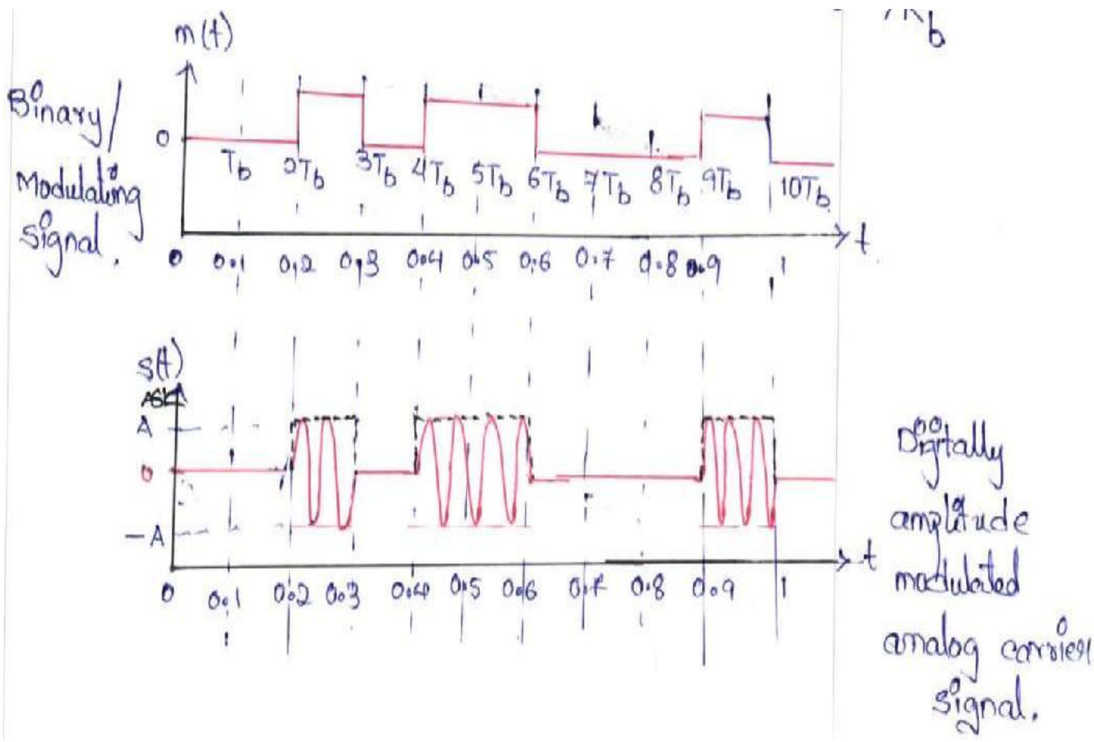
In coherent detection, the local carrier generated at the receiver is phase locked with the carrier at the transmitter .The coherent detection is synchronous detection.

2) Non Coherent Digital Modulation Techniques:

In Non coherent detection, detection process does not required receiver carrier to be phase locked with transmitter carrier.

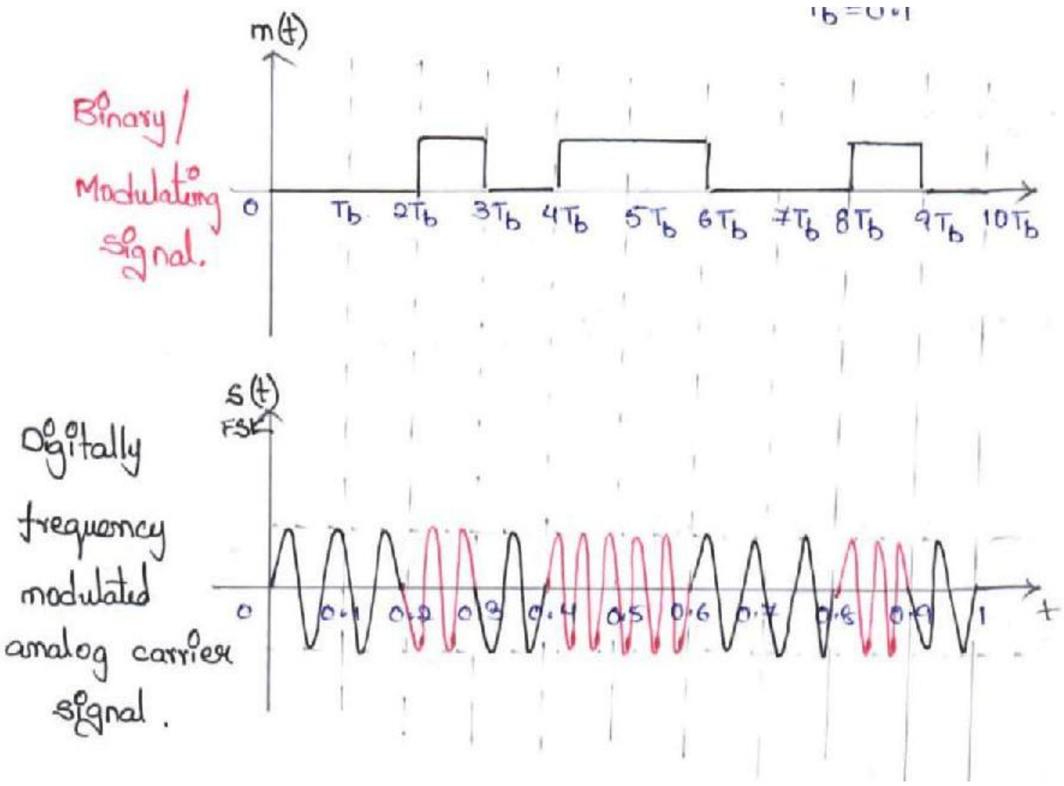
ASK-Amplitude shift keying (on – off keying):

→ Amplitude shift keying (on – off keying) – frequency is kept constant, amplitude has 2 levels (for bit 1 & for bit 0)
→ Principle of ASK:
For bit 1 transmit $S_1(t) = A \sin(2\pi f c t + 0)$ $0 \le t \le T_b$
bit 0 transmit $S_2(t) = 0$
→ T_b : Bit duration (duration of pulse) $T_b = 1/R_b$



FSK-Frequency shift keying:

→ Frequency shift keying.
Principle:
For bit 1 transmit $S_1(t) = A \sin(2\pi f c_1 t + 0)$ $0 \le t \le T_b$
For bit 0 transmit $S_2(t) = A \sin(2\pi f c_2 t + 0)$ $0 \le t \le T_b$
Bit duration $T_b = 1/R_b$ let $R_b = 10$; 0010110010
$T_b = 0.1$



PSK-phase shift keying:

→ phase shift keying

Principle:

For bit 1 transmit $S_1(t) = A \text{ Sin } (2\pi f_c t + \pi)$ $\quad 0 \le t \le T_b$

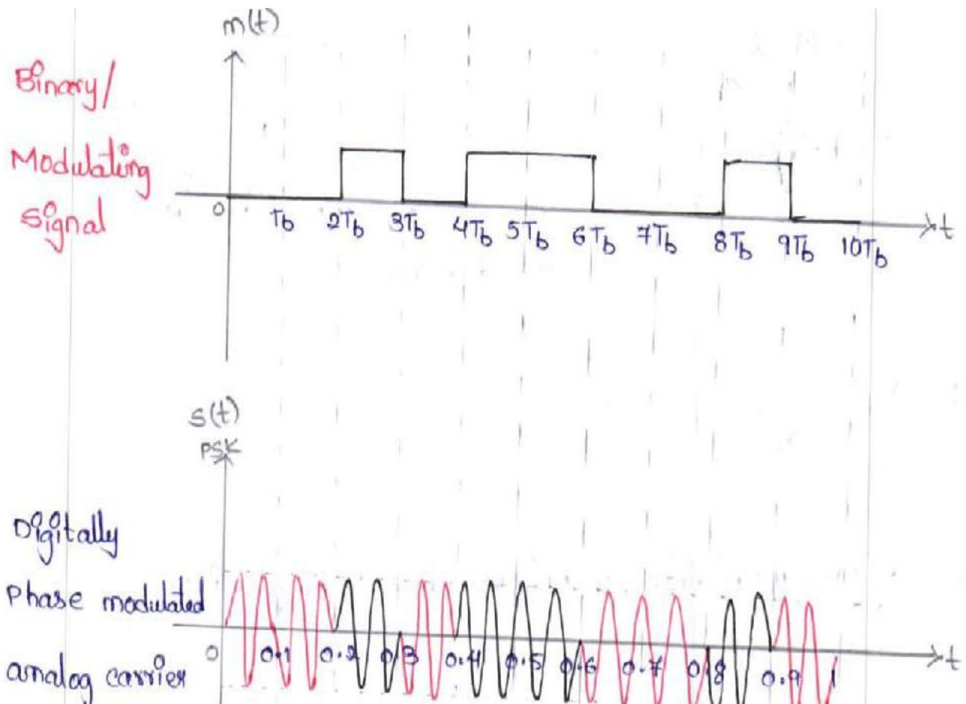
For bit 0 transmit $S_2(t) = A \text{ Sin } (2\pi f_c t + 0)$ $\quad 0 \le t \le T_b$

T_b : Bit duration = $1/R_b$

$R_b \rightarrow$ Bit Rate

Let R_b = 10 bps

Sequence → 0010110010



M-ary Modulation techniques:

Where $M = 2^k$

M : No. of symbols

K: No. of bits grouped into symbol.

Case (i) :

K = 1

$\Rightarrow M = 2$

Symbols : 0 $\Rightarrow T_b = T_s$

T_S = Symbol duration

T_b = Bit duration

$\rightarrow R_b = \frac{1}{T_b} = \frac{1}{T_S} = R_S$

M – ary digital modulation techniques are classified into :

1) M – ary ASK
2) M- ary FSK
3) M – ary PSK

$\rightarrow$ In binary ASK

$0 \rightarrow S_1(t) \quad 0 \leq t \leq T_b$

$1 \rightarrow S_2(t) \quad 0 \leq t \leq T_b$

$\rightarrow$ 2- ary digital modulation techniques are binary modulation techniques

Case (ii) :

K = 2

$\Rightarrow M = 4$

Symbols : 0 0

0 1 $\Rightarrow$ one symbol = 2 bits

1 0 $T_S = T_b \times 2$

1 1

$$\boxed{R_S = \frac{R_b}{K}}$$

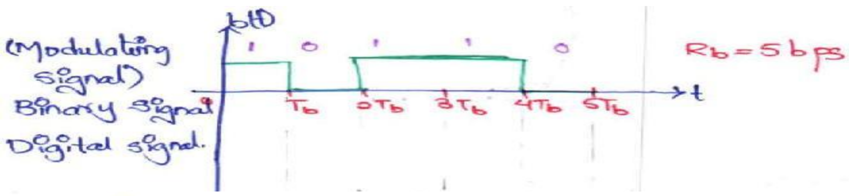
$\rightarrow$ 4 Level PSK is known as Quaternary PSK (QPSK).

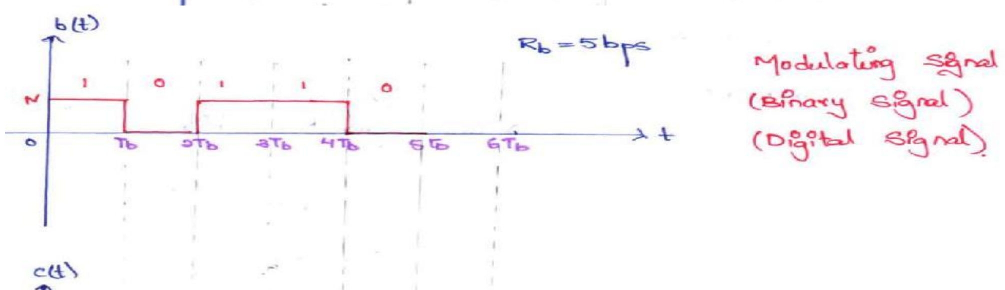
BINARY AMPLITUDE SHIFT KEYING (BASK):

Principle:

For bit 1 transmit $S_1(t)$ $A \cos 2\pi f_c t$ $\quad 0 \leq t \leq T_b$

For bit 0 transmit $S_2(t) = 0$ $\quad 0 \leq t \leq T_b$

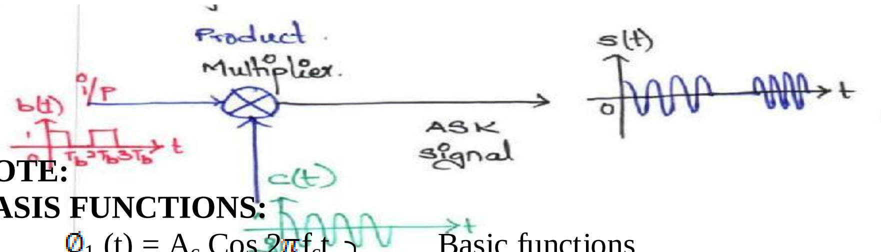




→ ASK Single is also known as interrupted Continuous Wave.
→ ASK is also known as on – off keying signal (00k)
→ Time domain description for ASK Signal.

$$\boxed{S_1(t) = b(t)\, A_c \cos 2\pi f_c t} \quad \rightarrow (1) \qquad 0 \le t \le T_b$$

GENERATION OF ASK:



NOTE:
BASIS FUNCTIONS:

$\emptyset_1(t) = A_c \cos 2\pi f_c t$
$\emptyset_1(t) = A_c \sin 2\pi f_c t$ } Basic functions used as carriers in communication

$$\boxed{P_S = \frac{A_c^2}{2} = \frac{E_b}{T_b}}$$

E_b: Ener... per bit per bit duration.
E_b = Bit energy.
T_b : Bit duration
P_S: power of signal.

$$\boxed{\Rightarrow A_C = \sqrt{\frac{2E_b}{T_b}} = \sqrt{2P_S}}$$

$1 \Rightarrow S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t \qquad 0 \le t \le T_b$

$0 \Rightarrow S_2(t) = 0 \qquad 0 \le t \le T_b$ principle.

∴ **Basis function:**

$\emptyset_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t$

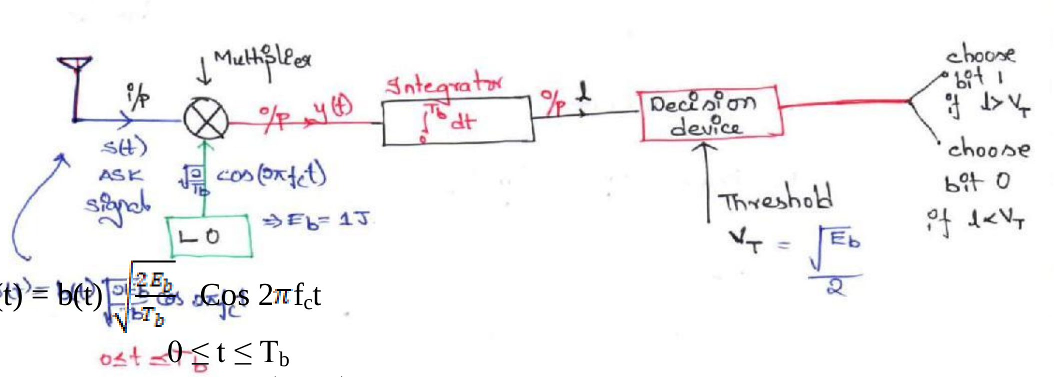
$\emptyset_1(t) = \sqrt{\frac{2E_b}{T_b}} \sin 2\pi f_c t$ } Orthogonal Basis functions

$\emptyset_2(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$ } Orthogonal Basis functions

$\emptyset_2(t) = \sqrt{\frac{2}{T_b}}$ Sin $2\pi f_c t$ $\qquad$ $E_b = 1$ J

$R_b = \frac{1}{T_b}$ $R_S = \frac{1}{T_S}$

DEMODU IGNAL: (Detection).



$S(t) = b(t)\sqrt{\frac{2E_b}{T_b}}$ Cos $2\pi f_c t$

$0 \leq t \leq T_b$

RECEIVED SIGNAL (ASK)

$S(t) = b(t)\sqrt{\frac{2E_b}{T_b}}$ Cos $2\pi f_c t$ $\qquad$ $0 \leq t \leq T_b$

MULTIPLIER O/P:

$Y(t) = s(t)\sqrt{\frac{2}{T_b}}$ COs $2\pi f_c t$

ASK signal.

INTEGRATOR O/P:

$$l = \int_0^{T_b} y(t)dt$$

$$l = \int_0^{T_b} S(t)\ \sqrt{\frac{2}{T_b}}\ Cos\ 2\pi f_c t dt$$

$$= \sqrt{\frac{2}{T_b}} \int_0^{T_b} S(t)\ Cos\ 2\pi f_c t dt.$$

$$= \sqrt{\frac{2}{T_b}} \int_0^{T_b} b(t)\sqrt{\frac{2E_b}{T_b}}\ Cos 2\pi f_c t . Cos 2\pi f_c t\ dt$$

$$= \sqrt{\frac{2}{T_b}}\sqrt{\frac{2E_b}{T_b}} \int_0^{T_b} Cos^2 2\pi f_c t\ \ dt$$

$$= \frac{2}{T_b}\sqrt{E_b} \int_0^{T_b} \frac{1 + Cos 4\pi f_c t}{2}\ dt.$$

$$= \frac{2\sqrt{E_b}}{T_b}\left[\frac{t}{2} + \frac{Sin 4\pi f_c t}{Cos 4\pi f_c t}\right]_0^{T_b}$$

$$= \frac{2\sqrt{E_b}}{T_b}\left[\frac{T_b}{2} + \frac{Sin 4\pi f_c T_b}{2 \times 4\pi f_c}\right]$$

$$= \frac{\sqrt{E_b}}{T_b}\left[T_b + \frac{Sin 4\pi f_c T_b}{4\pi f_c}\right]$$

$$l\ = \frac{\sqrt{E_b}}{T_b}\left[T_b + Sin\ \frac{4\pi f_c T_b}{4\pi f_c}\right]$$

$f_c = \frac{n}{T_b} \longrightarrow$ no. of cycles of carrier.

$$l\ = \frac{\sqrt{E_b}}{T_b}\left[T_b + Sin\ \frac{4\pi \frac{n}{T_b} T_b}{4\pi f_c}\right]$$

$$= \frac{\sqrt{E_b}}{T_b}\left[T_b + \frac{\text{Sin } 4\pi \text{ n}}{4\pi\frac{n}{T_b}}\right]$$

$$= \frac{\sqrt{E_b}}{T_b}\left[T_b\left(1 + \frac{\text{Sin } 4\pi \text{ n}}{4\pi n}\right)\right]$$

$$= \sqrt{E_b} \;\therefore \text{Sin } 4\pi n = 0$$

DETECTION OF ASK SIGNAL USING NON- COHERENT DETECTOR:

i/P ASK → ENVELOPE DETECTOR

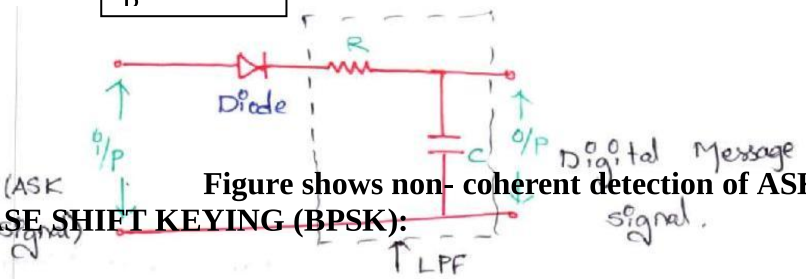


Figure shows non- coherent detection of ASK signal.

BINARY PHASE SHIFT KEYING (BPSK):

Principle:

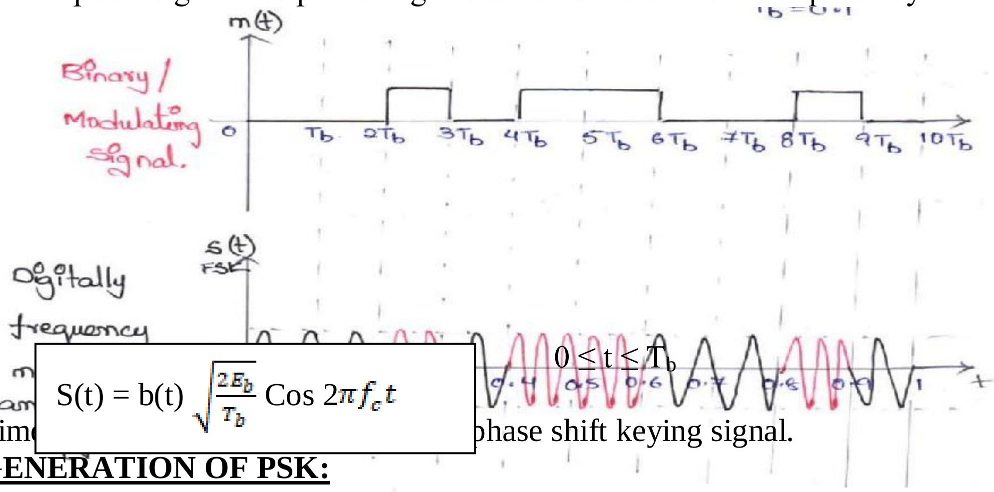
For bit "1" transmit $S_1(t) = \sqrt{\frac{2E_b}{T_b}}\text{ Cos }(2\pi f_c t + 0^0)$ $\quad 0 \le t \le T_b$

For bit "0" transmit $S_2(t) = \sqrt{\frac{2E_b}{T_b}}\text{ Cos }(2\pi f_c t + \pi)$ $\quad 0 \le t \le T_b$

$S_2(t) = \sqrt{\frac{2E_b}{T_b}}\text{ Cos }(2\pi f_c t)$ $\quad 0 \le t \le T_b$

→ PSK signals are known as antipodal signals.

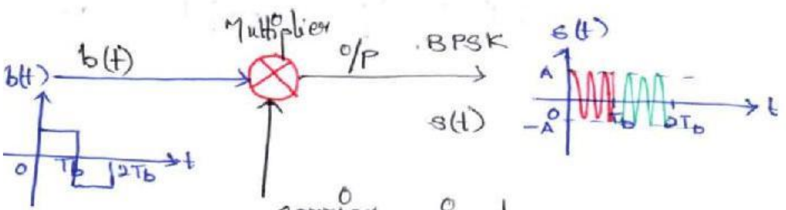
→ Antipodal signals are pair of signals which differs in terms of phase by 180^0



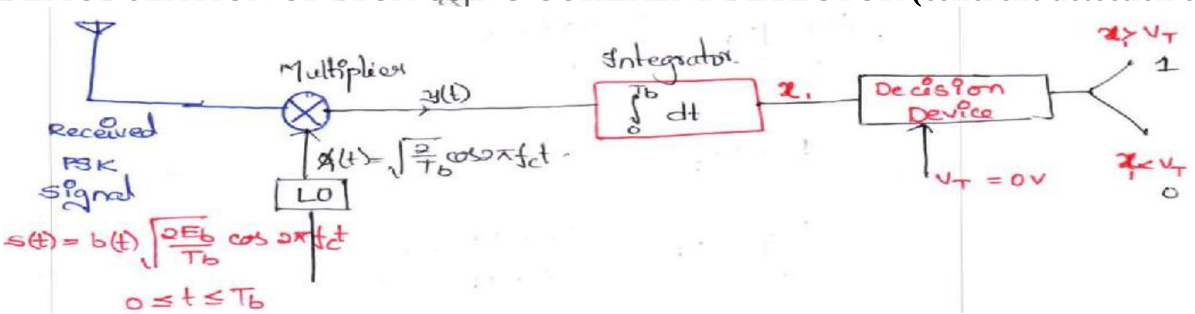
$S(t) = b(t)\sqrt{\frac{2E_b}{T_b}}\text{ Cos } 2\pi f_c t$ $\quad 0 \le t \le T_b$

Tim … hase shift keying signal.

GENERATION OF PSK:



DEMODULATION OF BPSK USING COHERENT DETECTOR (coherent detection of BPSK signals):



Received PSK:

S(t) = b(t) $\sqrt{\frac{2E_b}{T_b}}$Cos $2\pi f_c t$ $\quad 0 \le t \le T_b$

Let received signal is

S_1(t) = b(t) $\sqrt{\frac{2E_b}{T_b}}$ Cos $2\pi f_c t$ $\quad 0 \le t \le T_b$

MULTIPLIER o/p :

y(t) = $\sqrt{\frac{2E_b}{T_b}}$Cos $2\pi f_c t$ + $\sqrt{\frac{2}{T_b}}$ Cos $2\pi f_c t$

$= \frac{2}{T_b}\sqrt{E_b}\, Cos^2 2\pi f_c t$

INTEGRATOR O/P:

$$X_1 = \int_0^{T_b} y(t)\, dt$$

$$= \frac{2\sqrt{E_b}}{T_b}\frac{1}{2}\int_0^{T_b}\left(1 + Cos4\pi f_c t\right) dt$$

$$= \frac{\sqrt{E_b}}{T_b}\left[t + \frac{Sin4\pi f_c t}{4\pi f_c}\right]_0^{T_b}$$

$$= \frac{\sqrt{E_b}}{T_b}\left[T_b + \frac{Sin4\pi f_c T_b}{4\pi f_c}\right]$$

$$f_c = \frac{n}{T_b}$$

$$= \frac{\sqrt{E_b}}{T_b}\left[T_b + \frac{Sin4\pi n}{4\pi T_b}\, T_b\right]$$

$$X_1 = \sqrt{E_b}$$

$\Rightarrow$ $X_1 > V_T \Rightarrow \therefore$ Decision – Bit 1

→ Let the received signal is

S_2 (t) = - $\sqrt{\frac{2E_b}{T_b}}$ Cos $2\pi f_{c_1} t$ $\quad 0 \le t \le T_b$

$$X_1 = -\sqrt{E_b}$$

$\Rightarrow$ $X_1 > V_T \Rightarrow \therefore$ Decision – Bit 0

Note:

→ BPSK & BFSK are constant envelope digital modulation techniques.

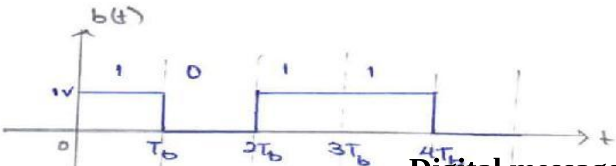
FSK:

→ frequency shift keying.

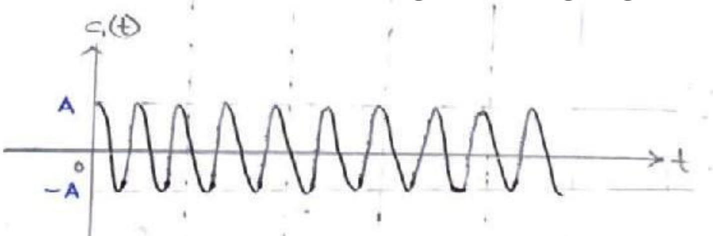
Principle:

For bit 1 transmit S_1(t) = $\sqrt{\frac{2E_b}{T_b}}$ Cos $(2\pi f_{c_1} t)$ $\quad 0 \le t \le T_b$

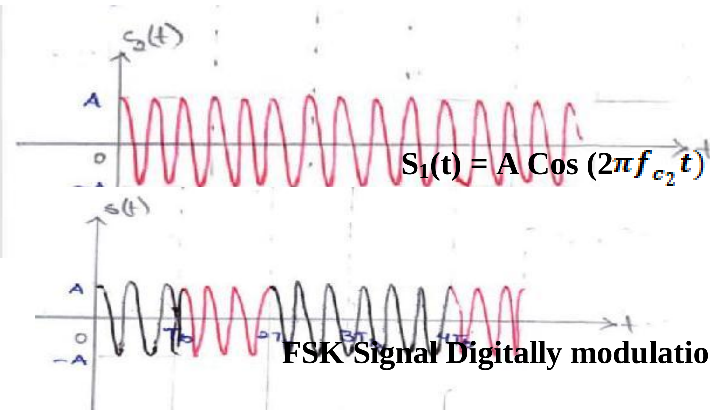
For bit 0 transmit S_2(t) = $\sqrt{\frac{2E_b}{T_b}}$ Cos $(2\pi f_{c_1} t)$ $\quad 0 \le t \le T_b$



Digital message signal



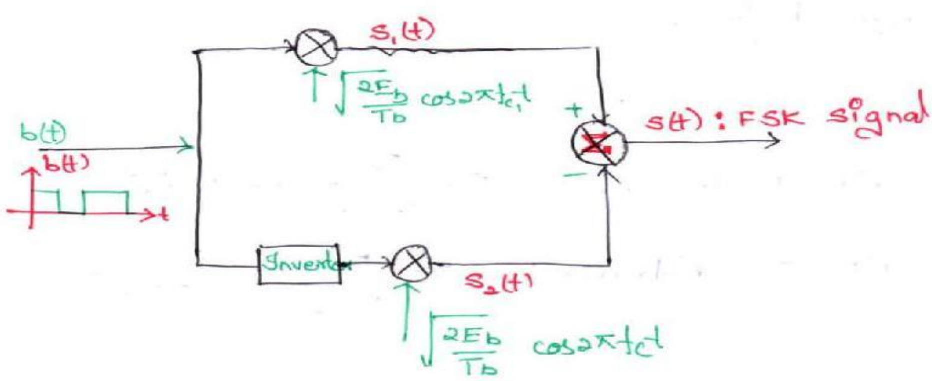
$S_1(t) = A \cos(2\pi f_{c_1} t)$



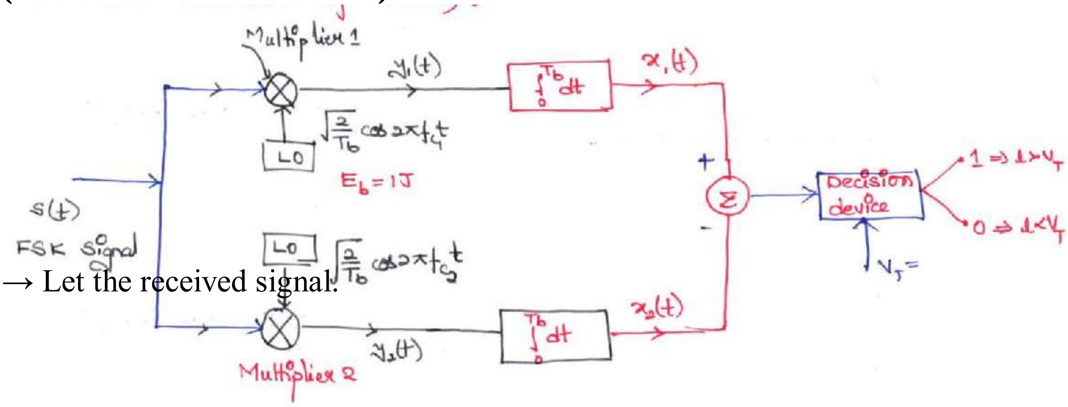
$S_1(t) = A \cos(2\pi f_{c_2} t)$

FSK Signal Digitally modulation signal.

GENERATION OF FSK:



DETECTION OF BINARY FSK SIGNAL:
(Coherent detection of FSK):



→ Let the received signal.

$S(t)$:FSK $= S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_{c_1} t)$ $\qquad 0 \le t \le T_b$

Multiplier 1 o/p:

$\Rightarrow y_1(t) - s_1(t).\ \sqrt{\frac{2}{T_b}} \cos 2\pi f_{c_1} t$

INTERGRATOR 1 O/P:

$\Rightarrow$ $x_1(t) = \frac{2}{T_b}\sqrt{E_b}\int_0^{T_b} Cos^2\, 2\pi f_{c_1} t dt$

$= \sqrt{E_b}$ ———————— (1)

Multiplier 2 o/p :

$\Rightarrow$ $y_2(t) – s_1(t). \sqrt{\frac{2}{T_b}}$ COs $2\pi f_{c_2} t$

Integrator 2 o/p:

$\Rightarrow$ $x_2(t) = \sqrt{\frac{2E_b}{T_b}}\int_0^{T_b} Cos 2\pi f_{c_1} t$ Cos $2\pi f_{c_2} t$ dt.

$= \frac{2\sqrt{E_b}}{2T_b}\int_0^{T_b} Cos 2\pi(f_{c_1} + f_{c_2})t + Cos\ 2\pi(f_{c_1} + f_{c_2})t$ dt.

$= \frac{2\sqrt{E_b}}{2T_b}\left[\frac{Sin\ 2\pi(f_{c_1} + f_{c_2})T_b}{2\pi(f_{c_1} + f_{c_2})} + \frac{Sin\ 2\pi(f_{c_1} + f_{c_2})T_b}{2\pi(f_{c_1} + f_{c_2})}\right]$

$f_{c_1} = f_{c_2} = n/T_b$

$= \frac{2\sqrt{E_b}}{2T_b}\left[\frac{Sin\ 4\pi n}{4\pi n} + \frac{Sin\ 4\pi n}{4\pi n}\right]$

$= 0$ $\rightarrow$ (2)

Summer o/p:

$l = x_1(t) – x_2(t)$

$= \sqrt{E_b}$ - 0

$l = \sqrt{E_b}$

Decision device O/P:

$\Rightarrow$ 1

$\therefore l > V_T$

$\rightarrow$ let the received signal be

$S_{FSK}(t) = S_2(t) = \sqrt{\frac{2E_b}{T_b}}$ Cos $2\pi f_{c2} t$ $\quad 0 \le t \le T_b$

Integrator 1 O/P:

$X_1(t) = 0$ $\longrightarrow$ (1)

Integrator 2 o/p:

$X_2(t) = \sqrt{E_b}$ $\longrightarrow$ (2)

Summer o/p: $l = x_1(t) = x_2(t)$\

$= -\sqrt{E_b}$

Decision device O/P :

$\Rightarrow$ 0

$\therefore l < V_T$

SPECTRAL ANALYSIS OF BINARY PSK:

S(t) PSK	$= b(T)\ A_C$ Cos $2\pi f_c t$

Time domain representation of PSK signal.

$\rightarrow$Apply Fourier transform to obtain spectrum of PSK signal.

$F\{ s(t)\} = s(f) = \int_{-\infty}^{\infty} s(t).e^{-j\omega t}\ dt$

$= \int_{-\infty}^{\infty}(b(t)A_c\ Cos\&2\pi f_c t)\ e^{-j2\pi f t}\ dt.$

$= A_c\ \int_{-\infty}^{\infty}(b(t)Cos\&2\pi f_c t)e^{-j2\pi f t}\ dt.$

Frequency shifting theorem.

B(f) = F{b(t)}

$\frac{1}{2}$ $[b(f - f_c) + b(f + f_c)]$ = F { b(t) Cos $2\pi f_c$t}

Spectrum of PSK $\boxed{S(t) = \frac{A_c}{2}\ [b(f - f_c) + b(f + f_c)]}$ frequency domain representation of PSK signal.

$\therefore$ s(t) = $A_C \int_{-\infty}^{\infty}$

$= \frac{A_c}{2} \int_{-\infty}^{\infty} b(t)\, .\, e^{-J2\pi(f-f_c)t}\ dt + \frac{A_c}{2} \int_{-\infty}^{\infty} b(t)\, .\, e^{-J2\pi(f+f_c)t}\ dt.$

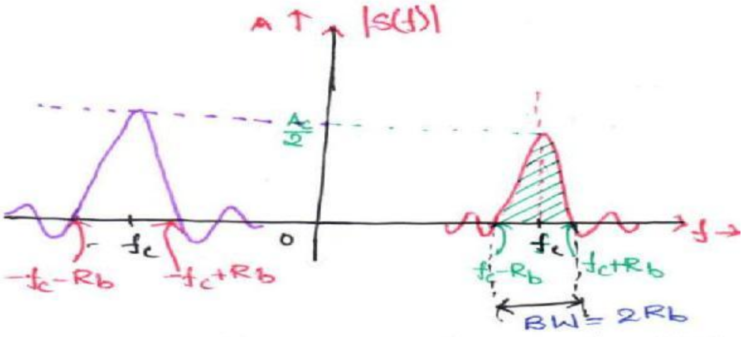


Figure shows spectrum of PSK signal.

→ BW of ASK signal is

BW = 2 R_b

SPECTRAL ANALYSIS OF BINARY ASK:

$\boxed{S(t) = b(t)\, A_C \text{ Cos } 2\pi f_c t}$ OF Binary ASK. Time domain representation ASK

→ Apply Fourier transforms to obtain spectrum of Ask signal.

F {s(t)} = s(f) = $\int_{-\infty}^{\infty} s(t) e^{-j\omega t}\ dt$

$= A_C \int_{-\infty}^{\infty} b(t) Cos\ 2\pi f_c t\ e^{-j\omega t}\ dt$

$= \frac{A_C}{2} [b(f - f_c) + b(f + f_c)]$

SPECTRAL ANALYSIS O F FSK SIGNAL:

<u>Principle:</u>

For bit '0' transmit $S_2(t) = A_c$ Cos $(2\pi f_{c_2} t + 0)$ $0 \leq t \leq T_b$

bit '1' transmit $S_1(t) = A_c$ Cos $(2\pi f_{c_1} t + 0)$ $0 \leq t \leq T_b$

→ To obtain spectrum.

Apply Fourier Transform

F[s,(t)] = $S_1(f) = \int_{-\infty}^{\infty} S_1 . e^{-j\omega t}\ dt$

$= \int_{-\infty}^{\infty} b(t) . A_c Cos\ (2\pi f_{c_1} t)\ e^{-j\omega t}\ dt.$

$$S_1(f) = \frac{A_c}{2}\ [\, b\ (f - f_{c_1}) + b(f + f_{c_1})]$$

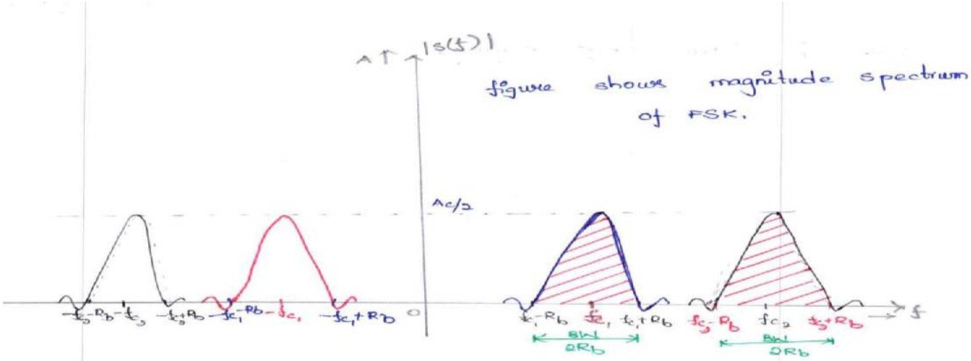
|||ly

$$F[s_2\ (t)] = S_2(f) = \frac{A_c}{2}\ [\, b\ (f - f_{c_2}) + b(f + f_{c_2})]$$

Spectrum of a FSK signal:

$$S_1(f) = \frac{A_c}{2}\ [\, b\ (f - f_{c_1}) + b(f + f_{c_1})]$$

$$S_2(f) = \frac{A_c}{2}\ [\, b\ (f - f_{c_2}) + b(f + f_{c_2})]$$



→ BW for FSK Transmission

$B_T = f_H - f_L$

$= (f_{c_2} + R_b) - (f_{c_1} - R_b)$

$= 2R_b + (f_{c_2} - f_{c_1})$

$$\boxed{B_T = 2R_b + \Delta f}$$

→ FS[…] width compared to BW required for ASK & PSK signal.

$BW_{FSK} > BW_{ASK}\ ,\ BW_{PSK}$

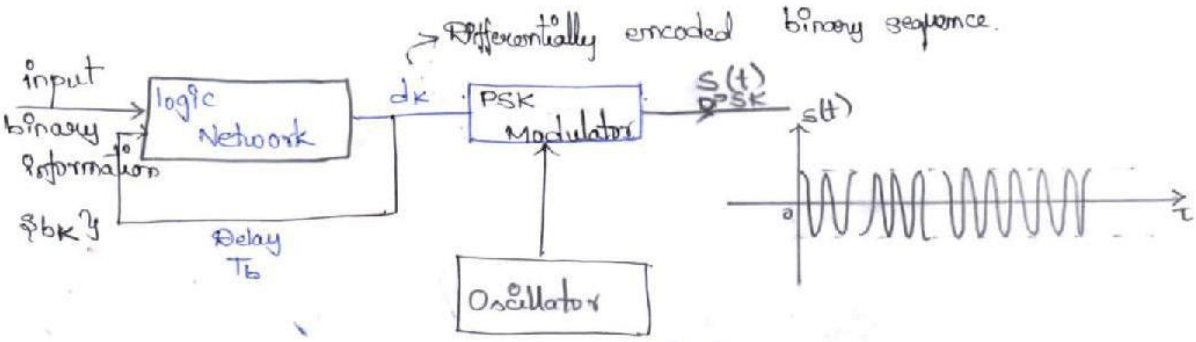
→ MSK → Minimum shift keying → used in GSM → Modified version of FSK.

DPSK- Differential phase shift keying:

→ Differential phase shift keying.

→ DPSK referred as " non coherent" version of PSK.

<u>DPSK Transmitter:</u>



$d_k = b_k\, d_{k-1} + \bar{b}_k\, \bar{d}_{k-1}$

$\{d_k\}$ binary information	1 0 1 1 0
d_{k-1}	1 1 0 0 0

d_k reference bit →	1 1 0 0 0 1
Phase of carrier transmitted	0^0 0^0 π π π 0^0

DPSK Receiver:

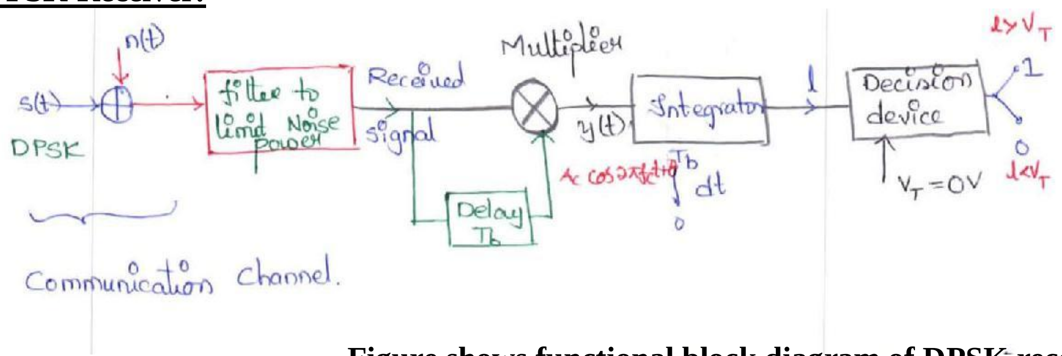


Figure shows functional block diagram of DPSK receiver.

Multiplier o/p:

$y(t) = s(t).A_c \, Cos 2\pi f_c t$

$= A_C^2 Cos^2 \, 2\pi f_c t$

INTEGRATOR O/P:

$$\int_0^{T_b} \frac{A_C^2}{2} \, (1 + Cos \, 4\pi f_c t) \, dt$$

$$= \frac{A_C^2}{2} \left[T_b + \frac{\text{Sin } 4\pi f_c t}{4\pi f_c} \right]_0^{T_b}$$

$$= \frac{A_C^2}{2} \left[T_b \frac{\text{Sin } 4\pi n / T_b \times T_b}{4\pi f_c} \right]$$

$$l = \frac{A_C^2}{2} T_b$$

o/p of decision device:

1) $l > V_T = OV$ ⇒ “1”

2) $\frac{A_C^2 T_b}{2} > V_T$ ⇒ “1”

3) $\frac{-A_C^2 T_b}{2} < V_T$ ⇒ “0”

4) $\frac{A_C^2 T_b}{2} > V_T$ ⇒ “1”

→ DPSK can be considered as non – coherent version PSK.

ADVANTAGES:

→ Receiver is simple.

DRAWBACK:

→ DPSK gives large bit error.

∴ Each bit recovered is dependent on previous bit.

QPSK-Quadrature phase shift keying:

(Quadrature phase shift keying) (off set QPSK)(4- level PSK)

(STAGGERED QPSK):

→ it is a M- ary PSK modulation technique.

$$M = 4$$
$$2^k = 4$$
$$K = 2$$

M → No. of symbols

K → No. of bits grouped into one symbol.

→ Symbols: 0 0 $\pi/4$: Phase $\emptyset = \frac{360^0}{M}$

0 1 $3\pi/4$

1 0 $5\pi/4$

1 1 $7\pi/4$

→ **Signals Transmitted**:

$S_1(t) = A_c \text{ Cos } (2\pi f_c t + \pi/4)$ $\quad 0 \le t \le T_s = 2T_b$

$S_2(t) = A_c \text{ Cos } (2\pi f_c t + 3\pi/4)$ $\quad 0 \le t \le 2T_b$

$S_3(t) = A_c \text{ Cos } (2\pi f_c t + 5\pi/4)$ $\quad 0 \le t \le 2T_b$

$S_4(t) = A_c \text{ Cos } (2\pi f_c t + 7\pi/4)$ $\quad 0 \le t \le 2T_b$

→ Generalized time domain description of QPSK

$S_i(t) = A_c \text{ Cos } [2\pi f_c t + (2i-1)\pi/4]$ $\quad 0 \le t \le T_s$

$i = 1, 2, 3\ldots..$

$$A_c = \sqrt{\frac{2E_s}{T_s}}$$

E_S : Energy of signal transmitted per symbol duration (symbol energy).

T_s : symbol duration

$S_i(t) = \sqrt{\frac{2E_s}{T_s}} \text{ Cos } [2\pi f_c t + (2i-1)\pi/4]$ $\quad 0 \le t \le T_s$

→ In QPSK:

$T_s = 2\,T_b$

$$R_S = \frac{1}{2T_b} = \frac{R_b}{2}$$

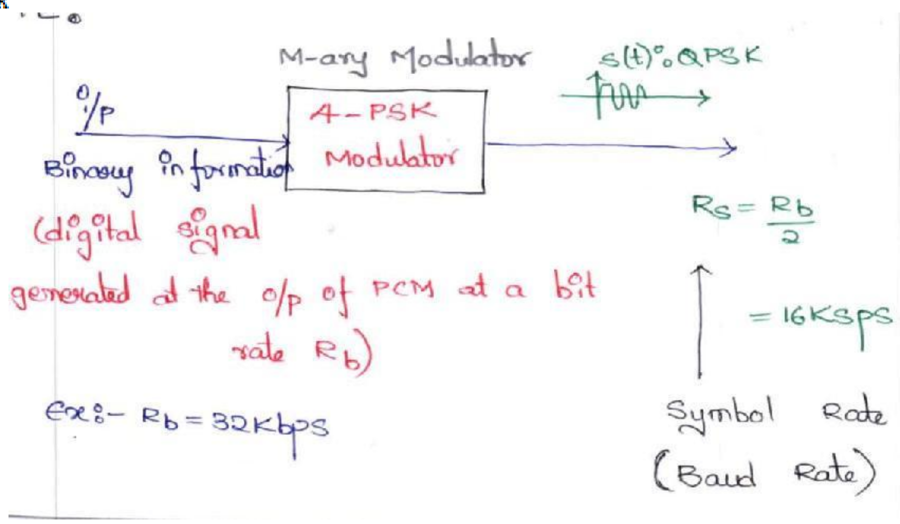
R_S: symbol rate

Is a reciprocal of symbol duration

→ In M – ary digital modulation technique

$R_s = \frac{R_b}{K}$

Note:



$S_i(t) = \sqrt{\frac{2E_s}{T_s}} \text{ Cos } [2\pi f_c t + (2i-1)\pi/4]$ $\quad 0 \le t \le T_s$

= $\sqrt{\frac{2E_s}{T_s}}$ [Cos ($2\pi f_c$t).Cos(2i – 1)$\pi/4$) – Sin ($2\pi f_c$t).Sin (2i -1) $\pi/4$]

i = 1 , 2, 3…..

= $\sqrt{\frac{2E_s}{T_s}}$ [Cos ($2\pi f_c$t).Cos(2i – 1)$\pi/4$) – = $\sqrt{\frac{2E_s}{T_s}}$ [Sin ($2\pi f_c$t).Sin (2i – 1)$\pi/4$)]

∴ Basis functions:

$\varphi_1(t) = \sqrt{\frac{2E_s}{T_s}}$ Cos $2\pi f_c t$

$\varphi_2(t) = \sqrt{\frac{2E_s}{T_s}}$ Sin $2\pi f_c t$

$$\boxed{\therefore S_i(t) = \varphi_1(t)\left(\pm\frac{1}{\sqrt{2}}\right) - \varphi_2(t)\left(\pm\frac{1}{\sqrt{2}}\right)}$$

QPSK receiver & transmitter:

BW of QPSK signal = $(f_c + R_S) - (f_c - R_S)$

$= 2R_S$

$= 2\frac{R_b}{2}$

BW = R_b

→ QPSK is a BW efficient and efficient digital modulation technique.

UNIT-V
Spread Spectrum Modulation

- Use of spread spectrum,
- direct sequence spread spectrum(DSSS),
- Code division multiple access,
- Ranging using DSSS Frequency Hopping spread spectrum,
- PN sequences: generation and characteristics,
- Synchronization in spread spectrum system,
- Advancements in the digital communication

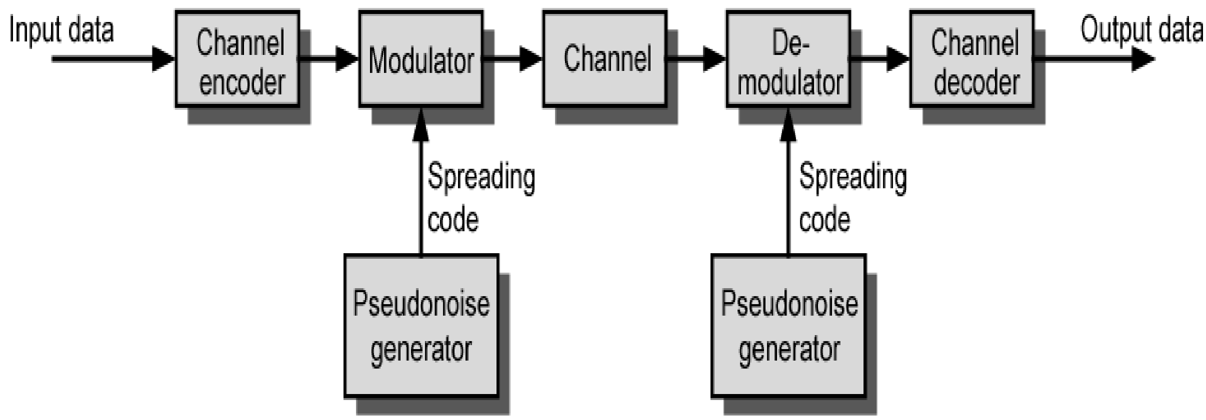
SPREAD SPECTRUM MODULATION

- Spread data over wide bandwidth
- Makes jamming and interception harder
- Frequency hoping
  - Signal broadcast over seemingly random series of frequencies
- Direct Sequence
  - Each bit is represented by multiple bits in transmitted signal
  - Chipping code

Spread Spectrum Concept:

- Input fed into channel encoder
  - Produces narrow bandwidth analog signal around central frequency
- Signal modulated using sequence of digits
  - Spreading code/sequence
  - Typically generated by pseudonoise/pseudorandom number generator
- Increases bandwidth significantly
  - Spreads spectrum
- Receiver uses same sequence to demodulate signal
- Demodulated signal fed into channel decoder

General Model of Spread Spectrum System:



Gains:

- Immunity from various noise and multipath distortion
  - Including jamming
- Can hide/encrypt signals
  - Only receiver who knows spreading code can retrieve signal
- Several users can share same higher bandwidth with little interference
  - Cellular telephones
  - Code division multiplexing (CDM)
  - Code division multiple access (CDMA)

Pseudorandom Numbers:

- Generated by algorithm using initial seed
- Deterministic algorithm
  - Not actually random
  - If algorithm good, results pass reasonable tests of randomness
- Need to know algorithm and seed to predict sequence

Frequency Hopping Spread Spectrum (FHSS):

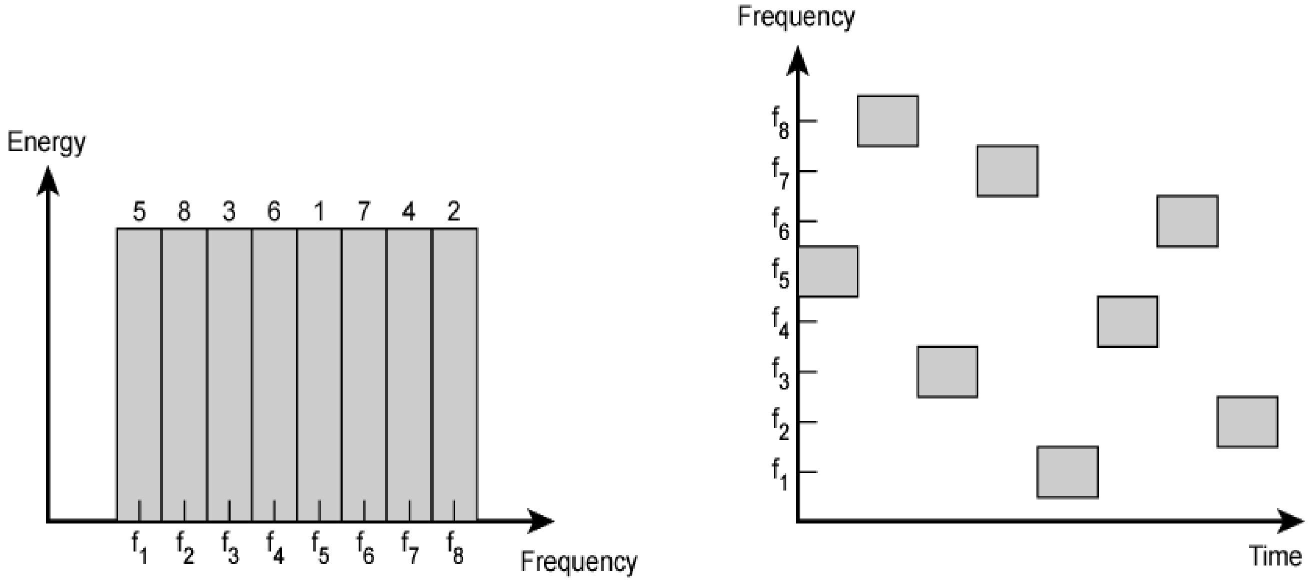
- Signal broadcast over seemingly random series of frequencies
- Receiver hops between frequencies in sync with transmitter
- Eavesdroppers hear unintelligible blips
- Jamming on one frequency affects only a few bits

Basic Operation:

- Typically 2k carriers frequencies forming 2k channels
- Channel spacing corresponds with bandwidth of input
- Each channel used for fixed interval

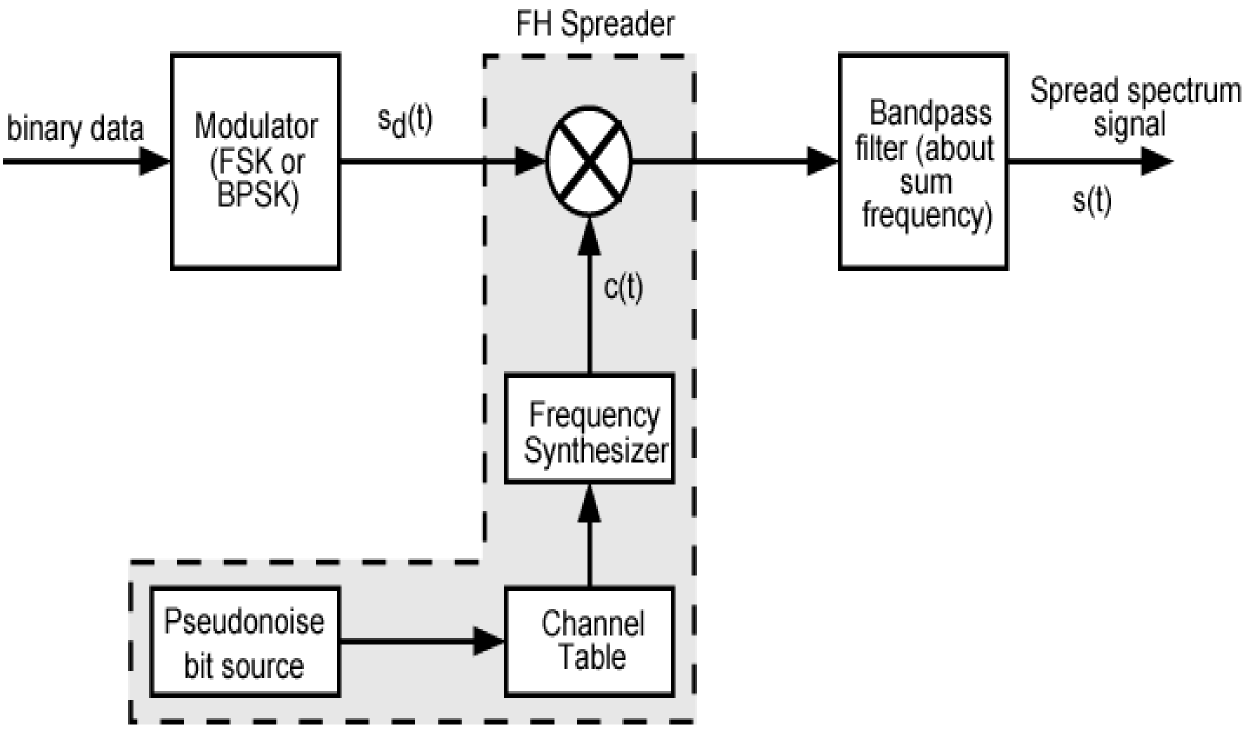
- 300 ms in IEEE 802.11
- Some number of bits transmitted using some encoding scheme
    - May be fractions of bit (see later)
- Sequence dictated by spreading code

Frequency Hopping Example:

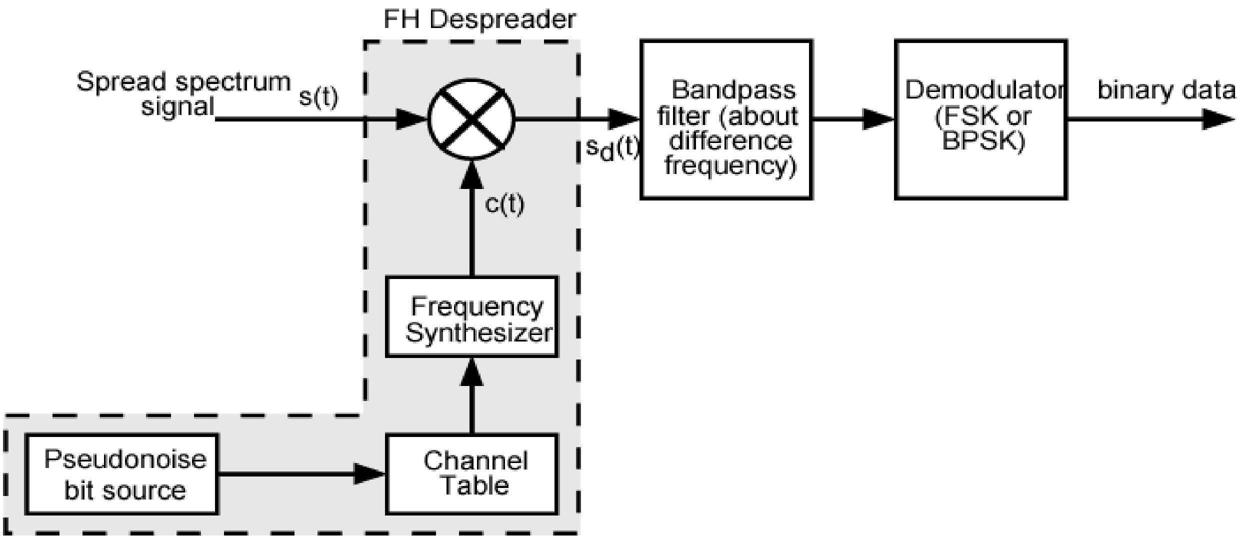


(a) Channel assignment (b) Channel use

Frequency Hopping Spread Spectrum System (Transmitter):



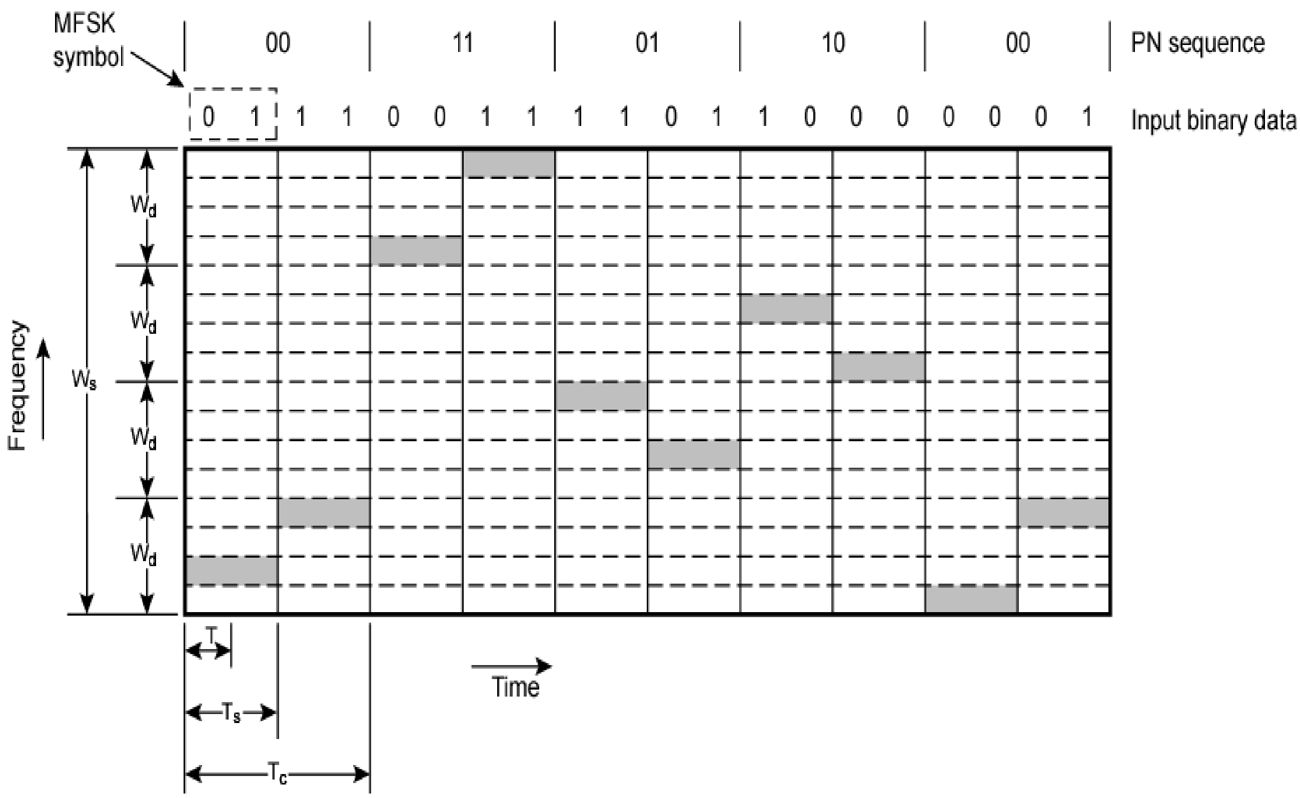
Frequency Hopping Spread Spectrum System (Receiver):



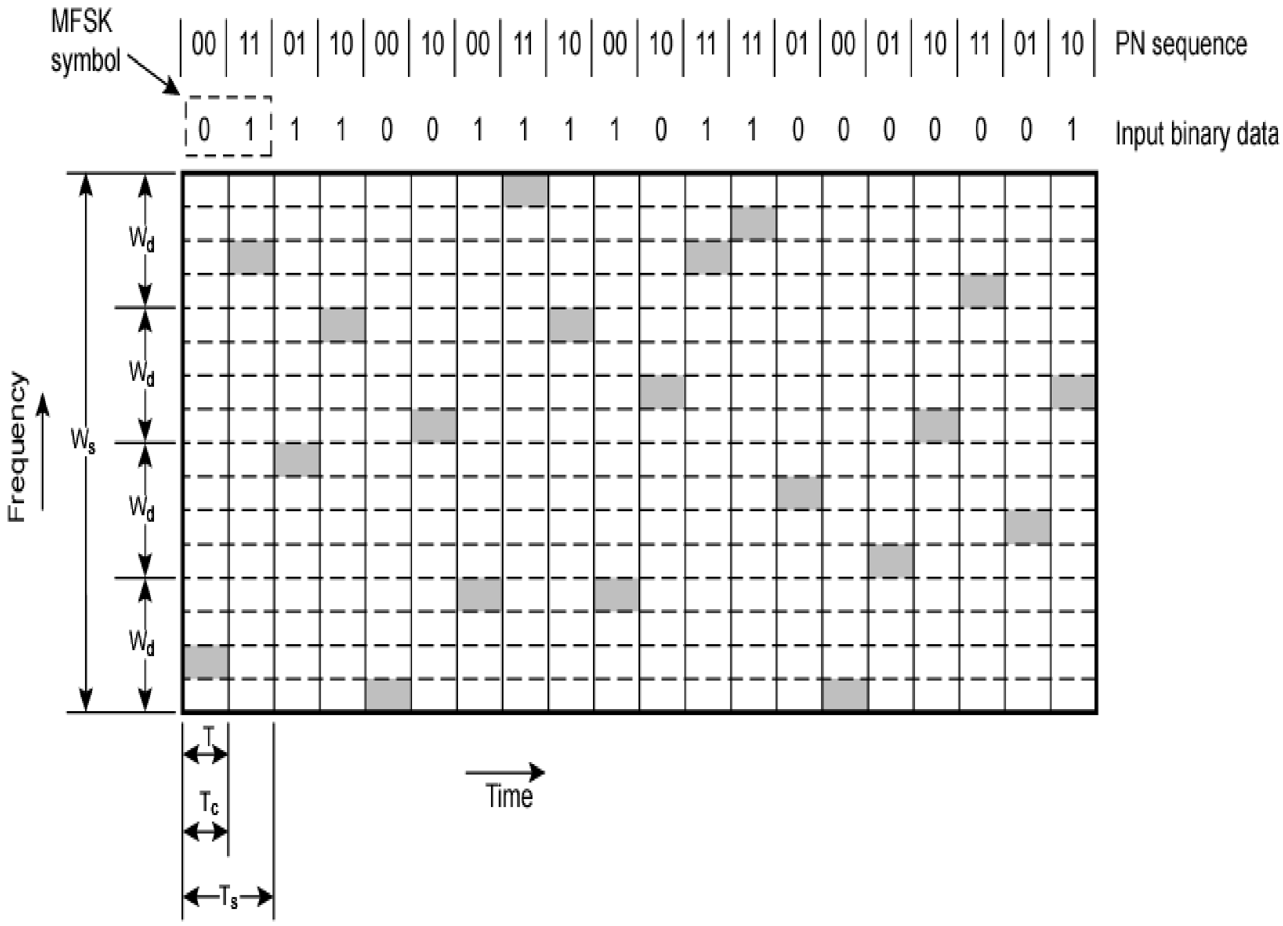
Slow and Fast FHSS:

- Frequency shifted every Tc seconds
- Duration of signal element is Ts seconds
- Slow FHSS has Tc ≥ Ts
- Fast FHSS has Tc < Ts
- Generally fast FHSS gives improved performance in noise (or jamming)

Slow Frequency Hop Spread Spectrum Using MFSK (M=4, k=2)



Fast Frequency Hop Spread Spectrum Using MFSK (M=4, k=2)



FHSS Performance Considerations:

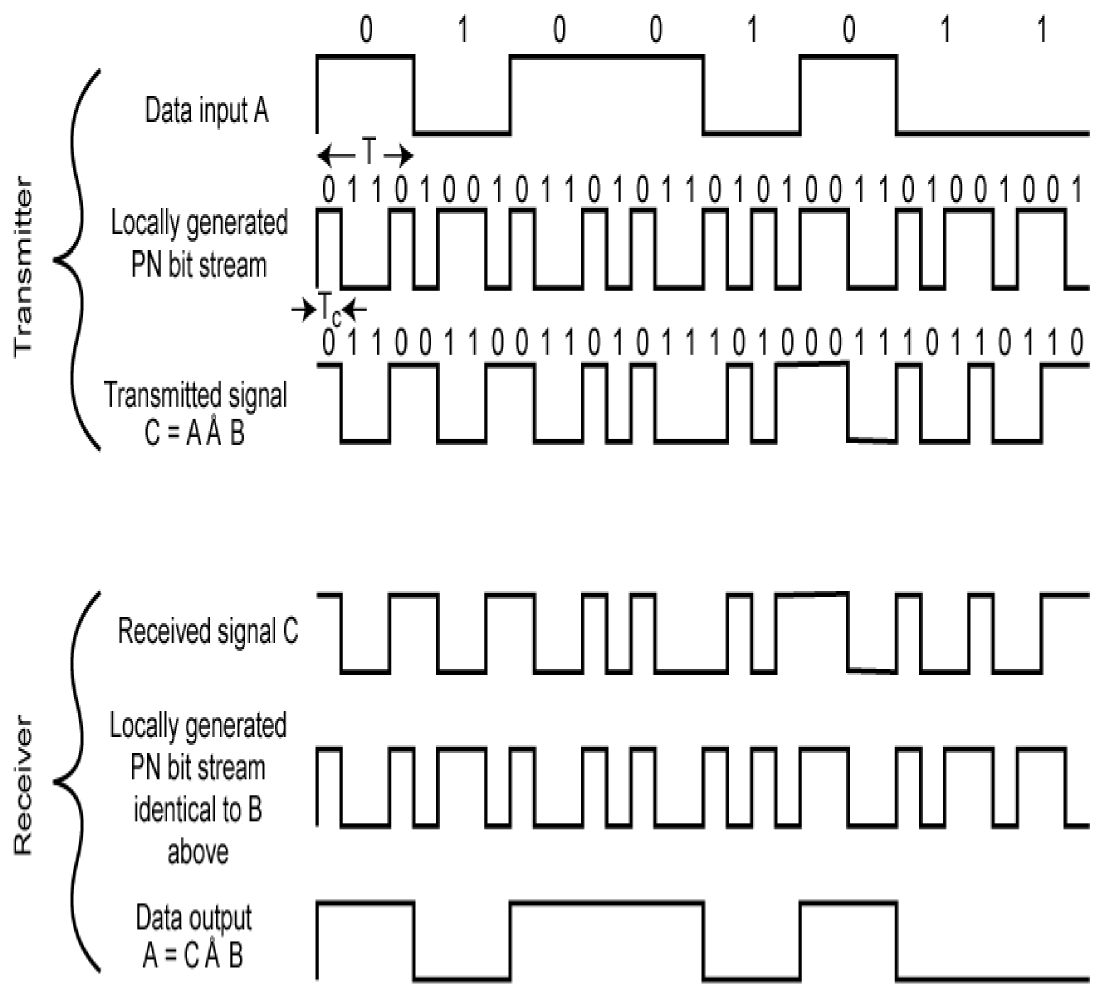
- Typically large number of frequencies used
  - Improved resistance to jamming

Direct Sequence Spread Spectrum (DSSS):

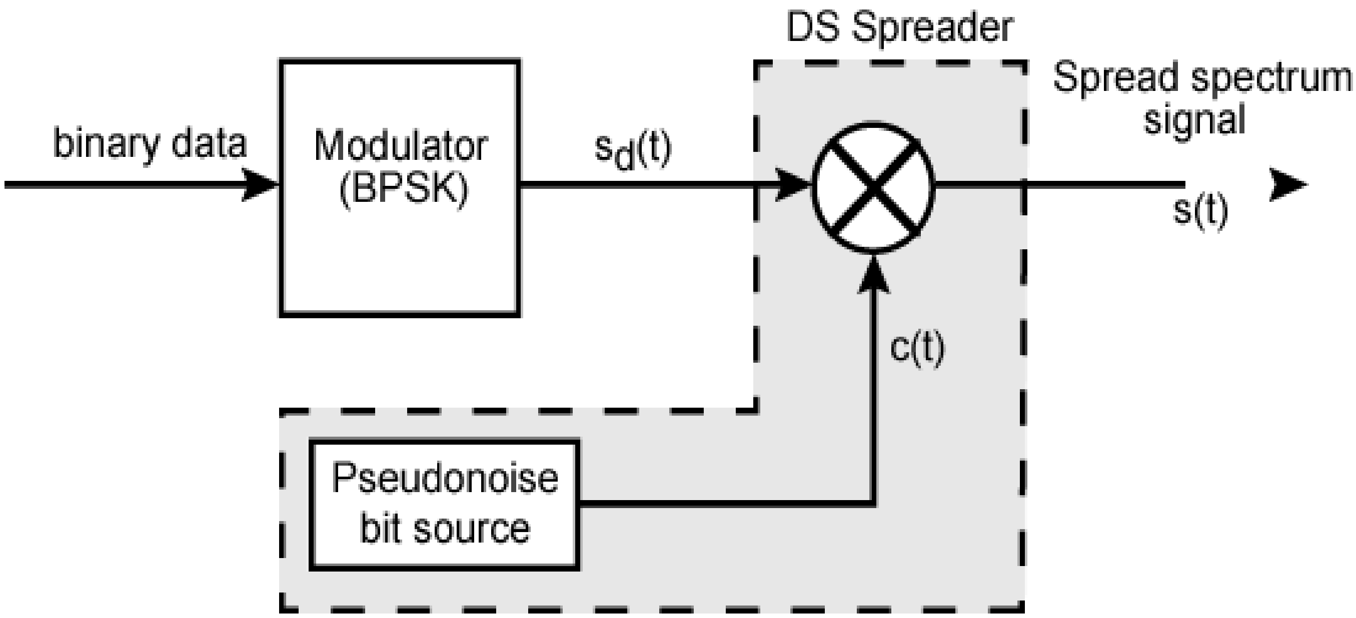
- Each bit represented by multiple bits using spreading code
- Spreading code spreads signal across wider frequency band
  - In proportion to number of bits used

  - 10 bit spreading code spreads signal across 10 times bandwidth of 1 bit code
- One method:
  - Combine input with spreading code using XOR
  - Input bit 1 inverts spreading code bit
  - Input zero bit doesn't alter spreading code bit
  - Data rate equal to original spreading code
- Performance similar to FHSS

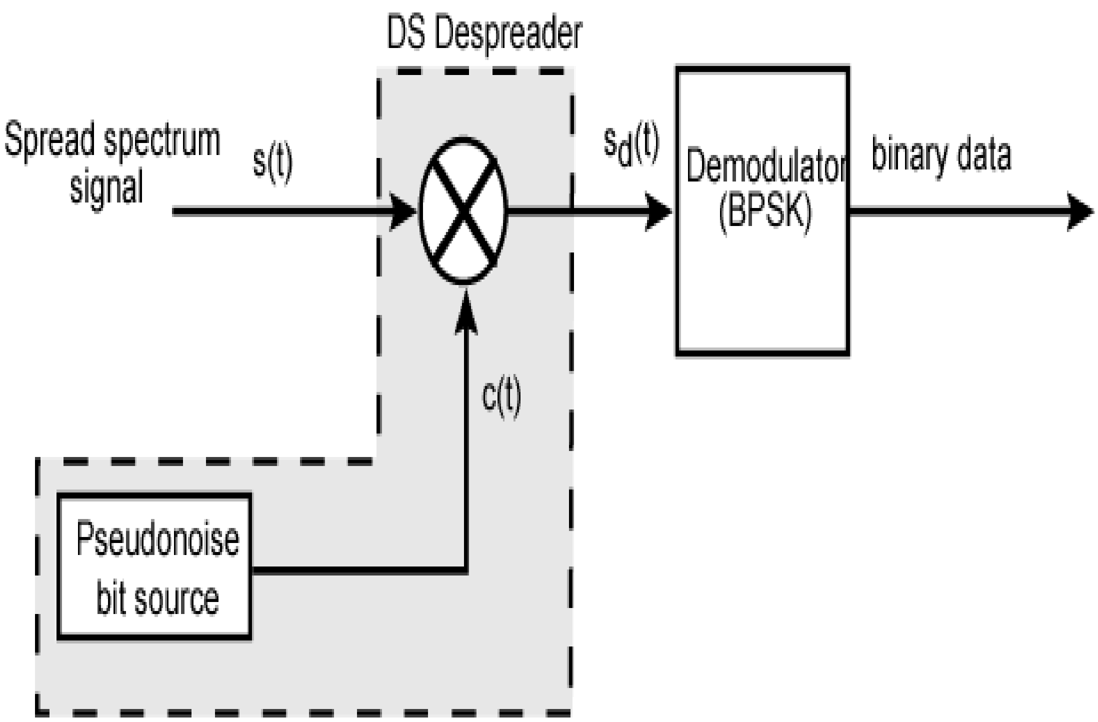
Direct Sequence Spread Spectrum Example:



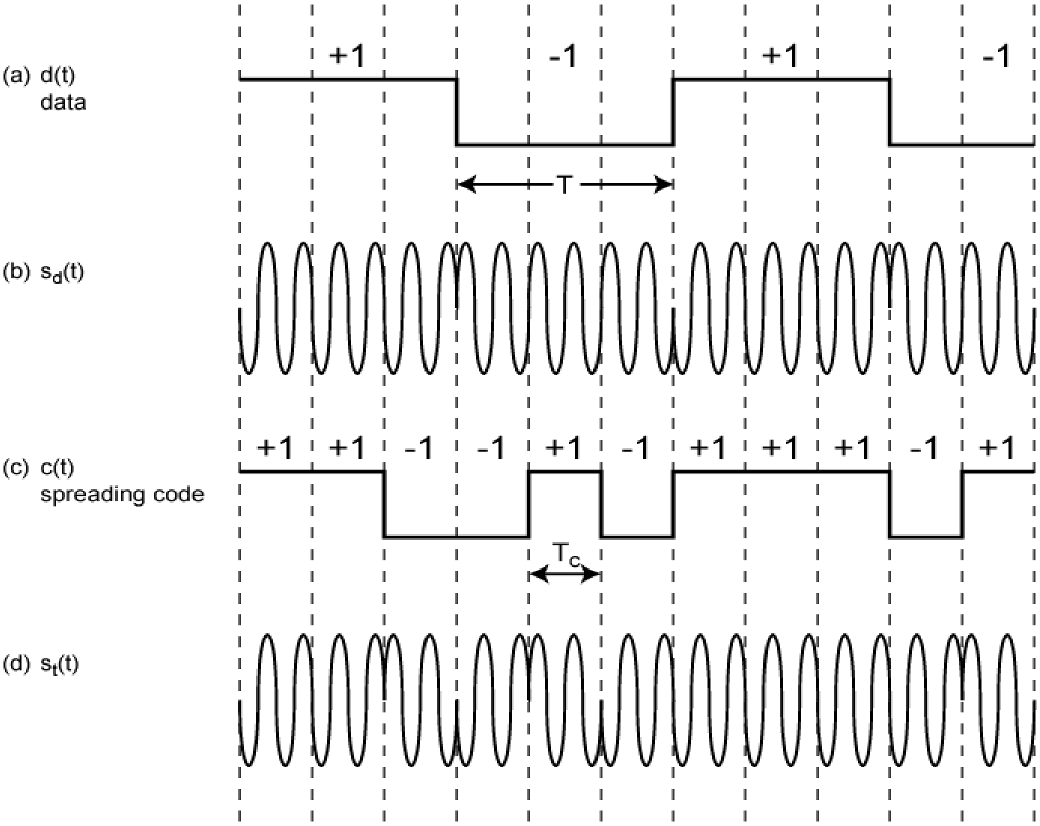
Direct Sequence Spread Spectrum Transmitter:



Direct Sequence Spread Spectrum Receiver:



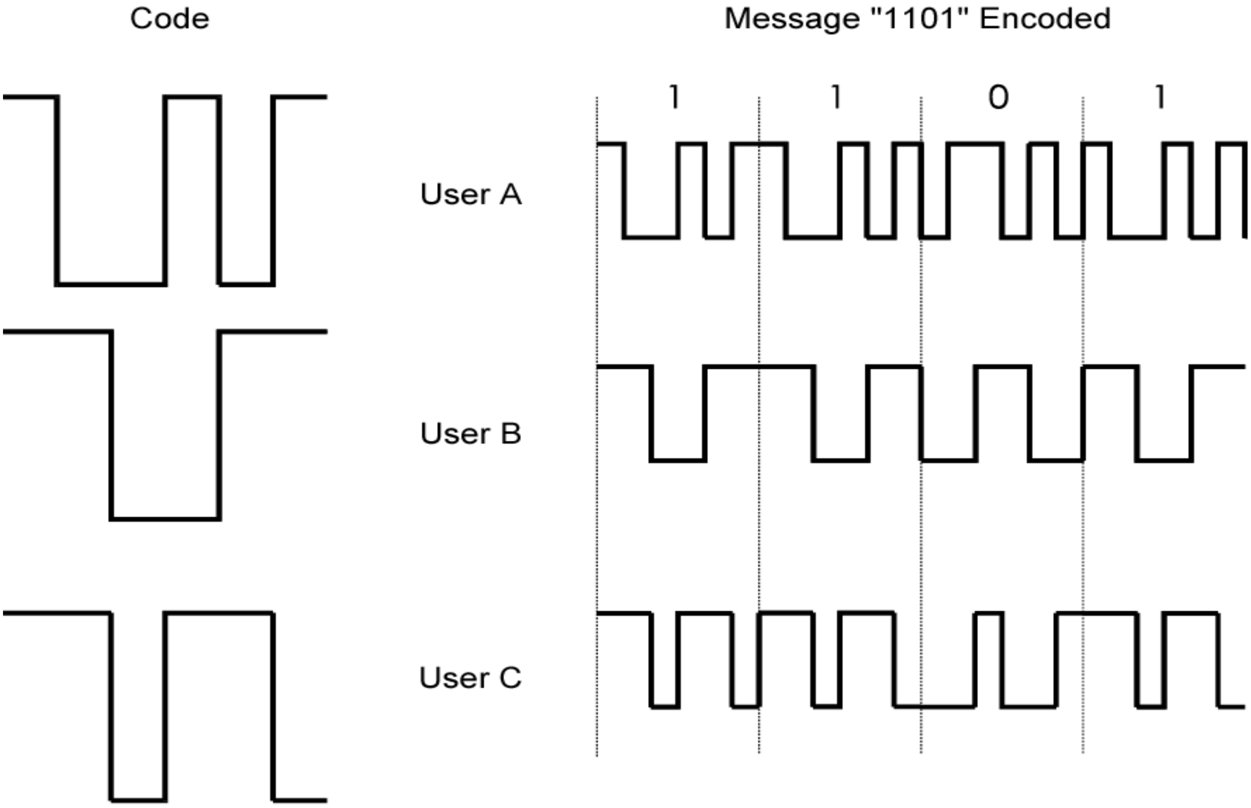
Direct Sequence Spread Spectrum Using BPSK Example:



Code Division Multiple Access (CDMA):

- Multiplexing Technique used with spread spectrum
- Start with data signal rate *D*
  - Called bit data rate
- Break each bit into *k* chips according to fixed pattern specific to each user
  - User's code
- New channel has chip data rate *kD* chips per second
- E.g. *k*=6, three users (A,B,C) communicating with base receiver R
- Code for A = <1,-1,-1,1,-1,1>
- Code for B = <1,1,-1,-1,1,1>
- Code for C = <1,1,-1,1,1,-1>

CDMA Example:

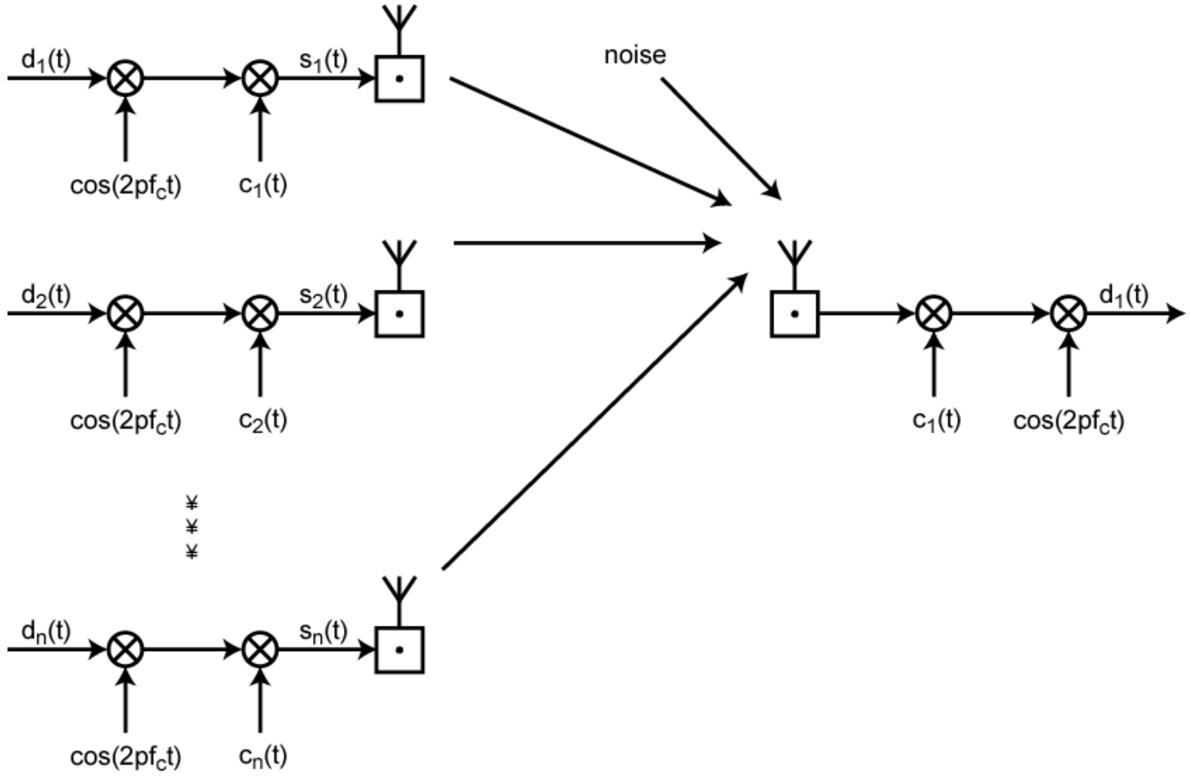


- Consider A communicating with base
- Base knows A’s code
- Assume communication already synchronized
- A wants to send a 1
  - Send chip pattern <1,-1,-1,1,-1,1>
    - A’s code
- A wants to send 0
  - Send chip[pattern <-1,1,1,-1,1,-1>
    - Complement of A’s code
- Decoder ignores other sources when using A’s code to decode
  - Orthogonal codes

CDMA for DSSS:

- n users each using different orthogonal PN sequence
- Modulate each users data stream
  - Using BPSK
- Multiply by spreading code of user

CDMA in a DSSS Environment:



d1(t)
s1(t)
cos(2pfct)
c1(t)
noise
d2(t)
s2(t)
cos(2pfct)
c2(t)
d1(t)
c1(t)
cos(2pfct)
¥
¥
¥
dn(t)
sn(t)
cos(2pfct)
cn(t)